Introduction to Interactive Decision Maps (IDM) method

Visualization of Pareto Frontier

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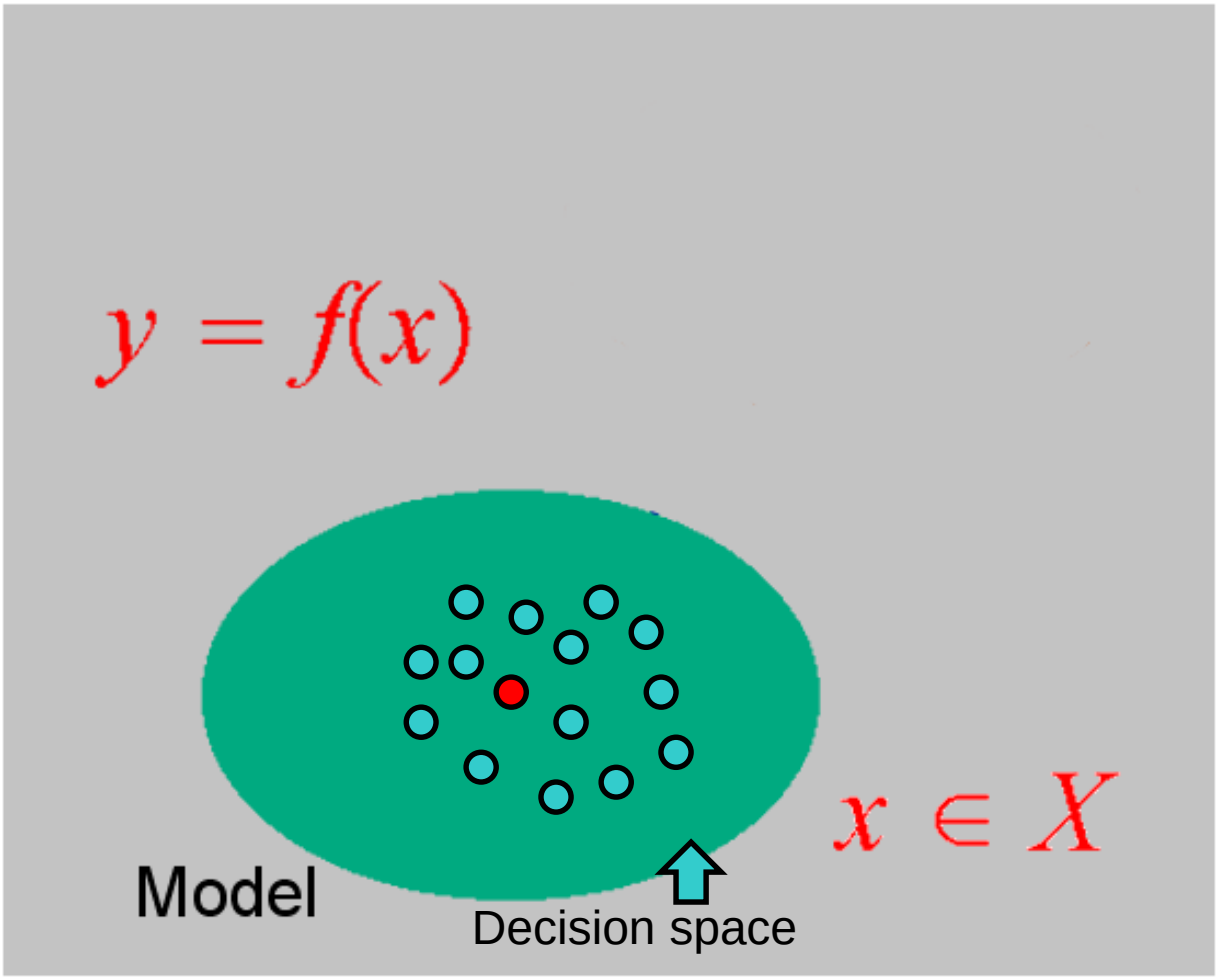
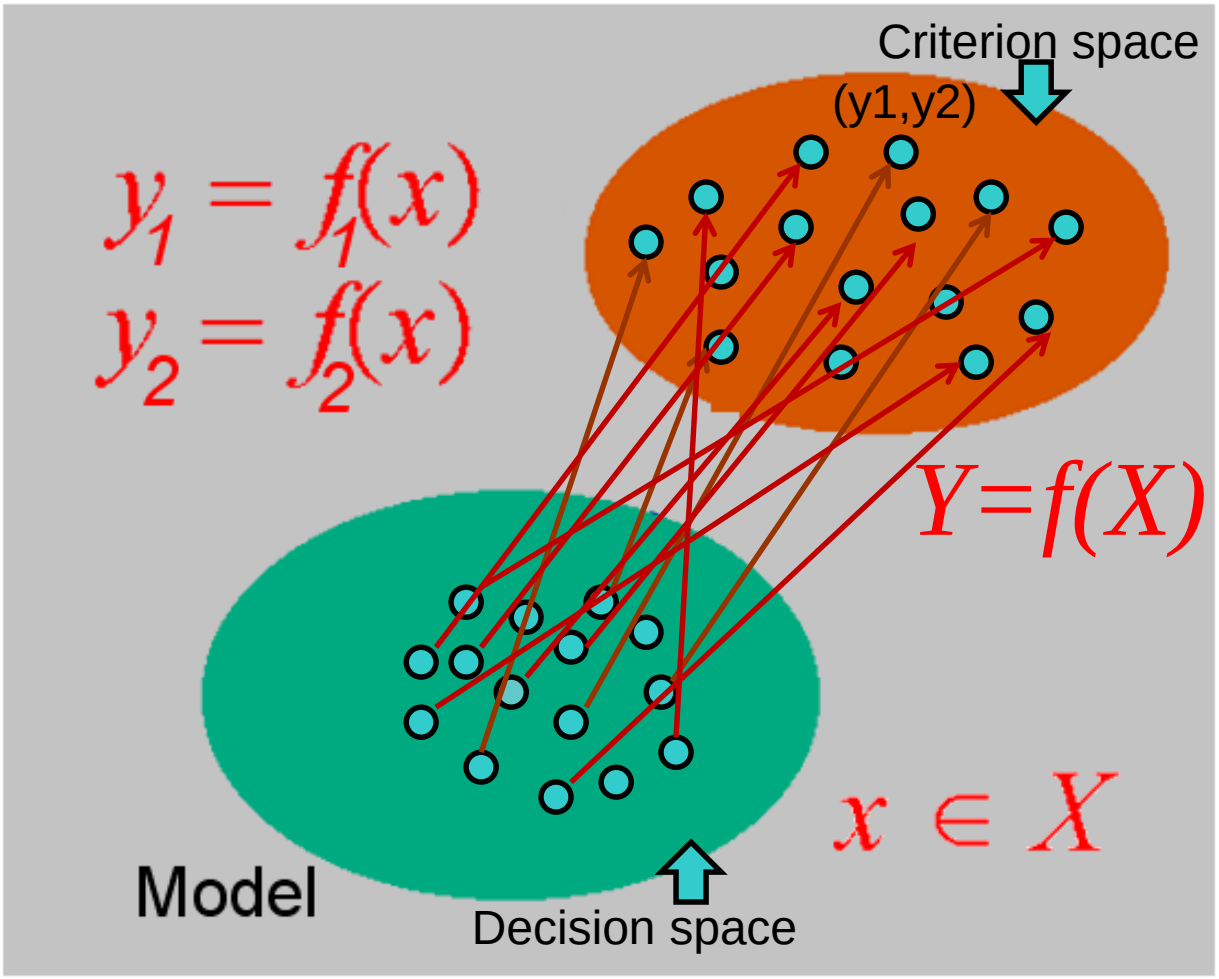
Zvolen, Slovakia, December 2, 2014

Few words concerning multi-objective optimization



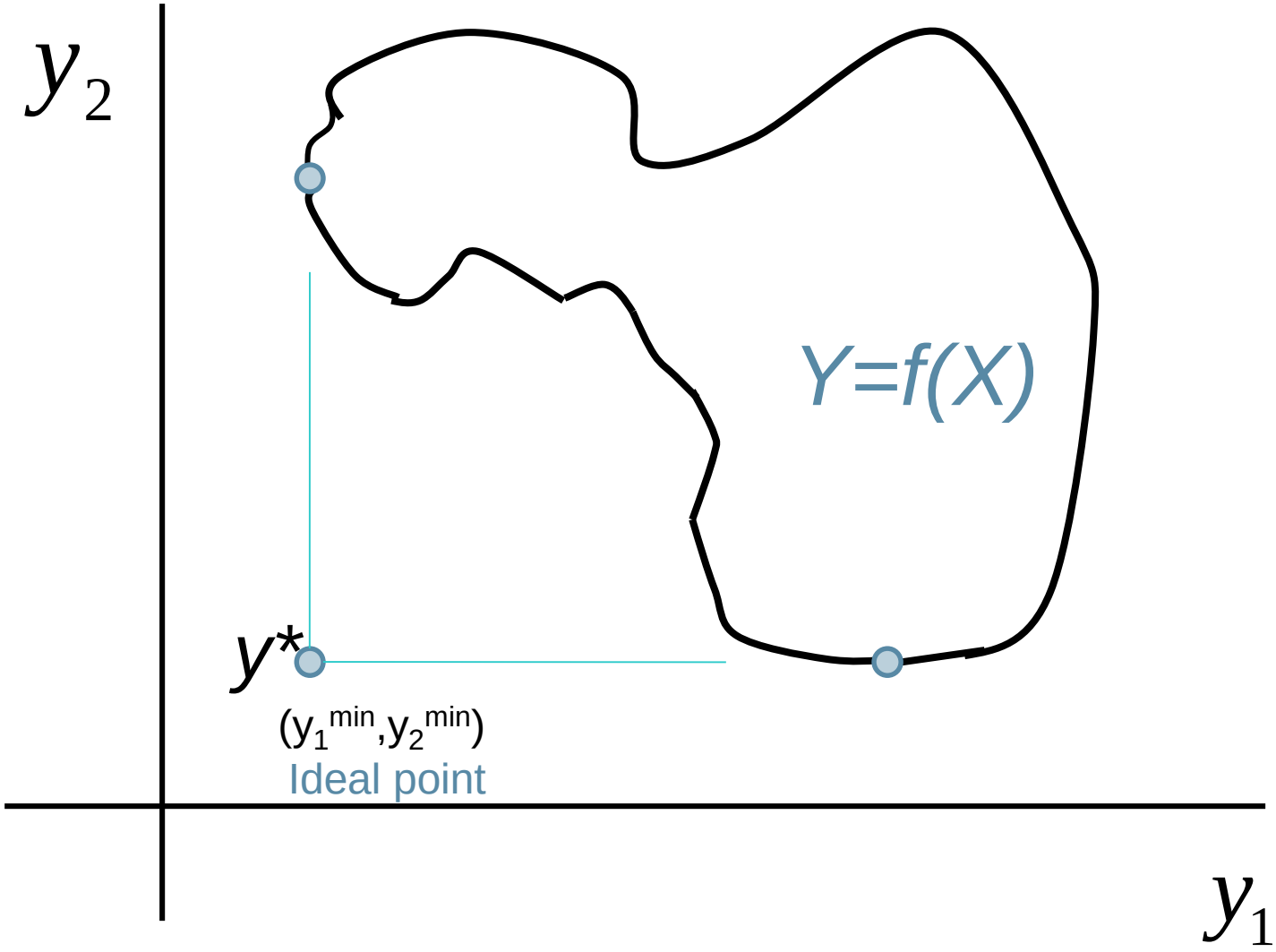
X - feasible set in decision space



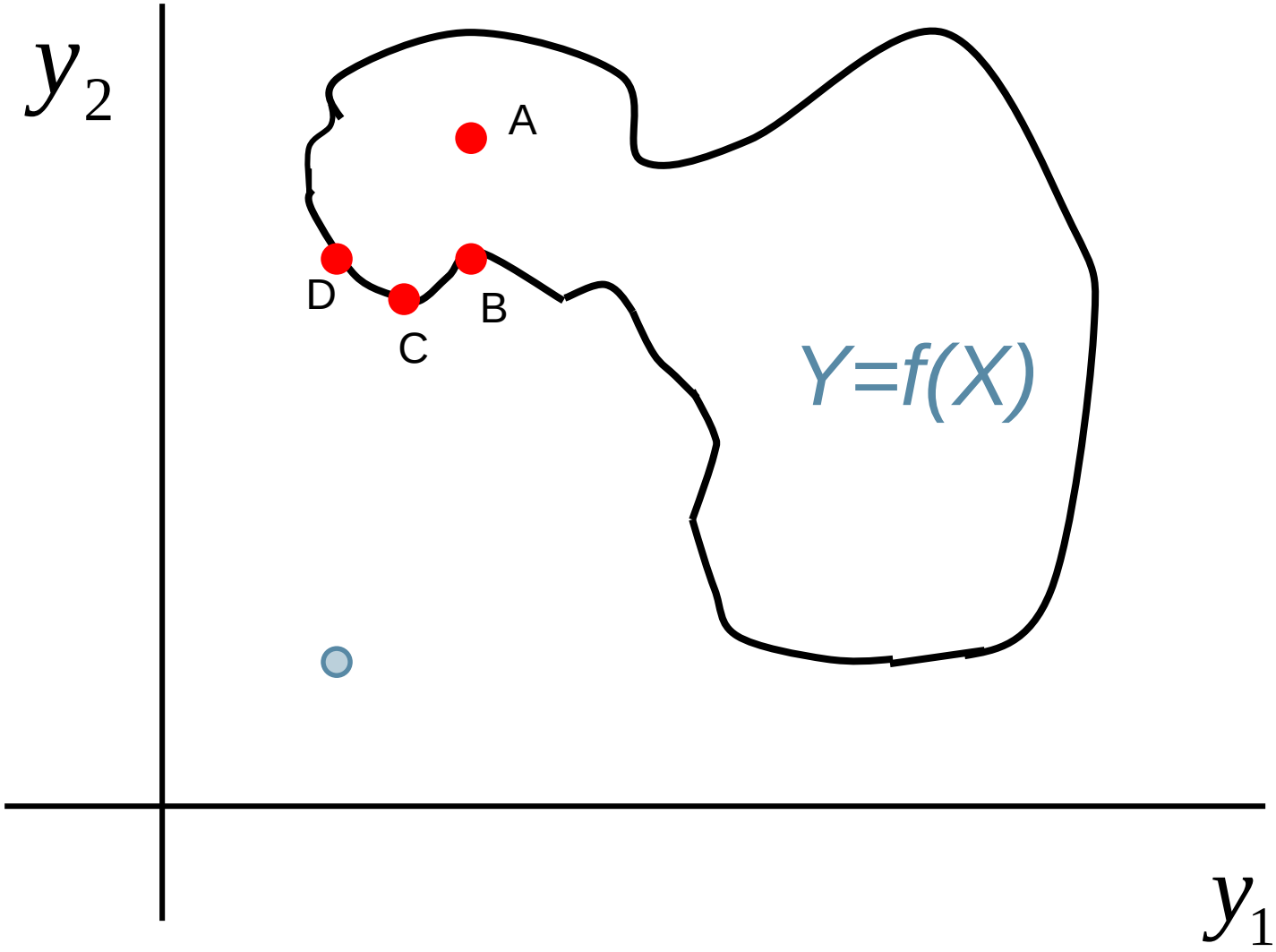
X - feasible set in decision space

Y=f(X) - feasible set in criterion space

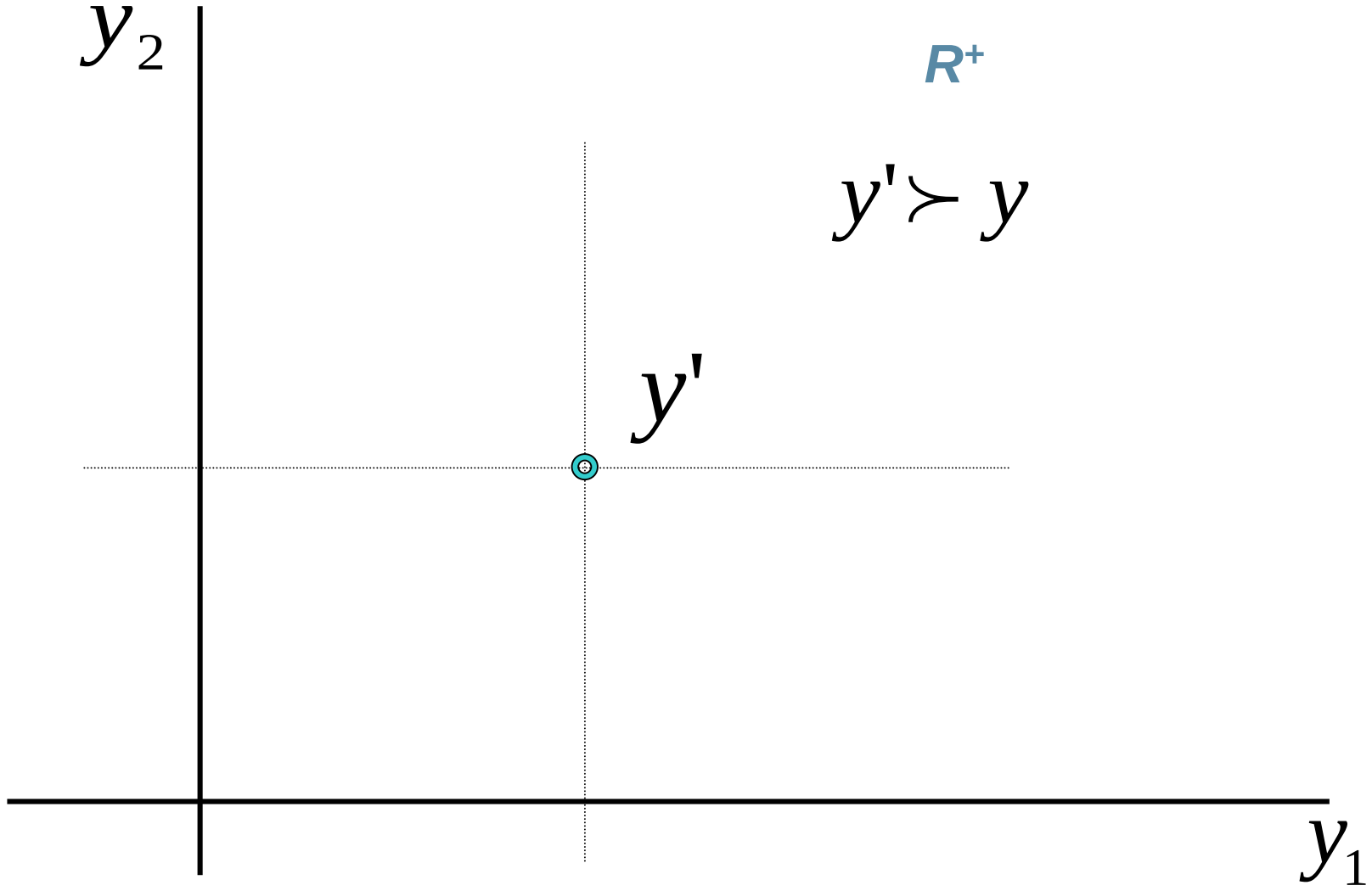
Feasible set in criterion space



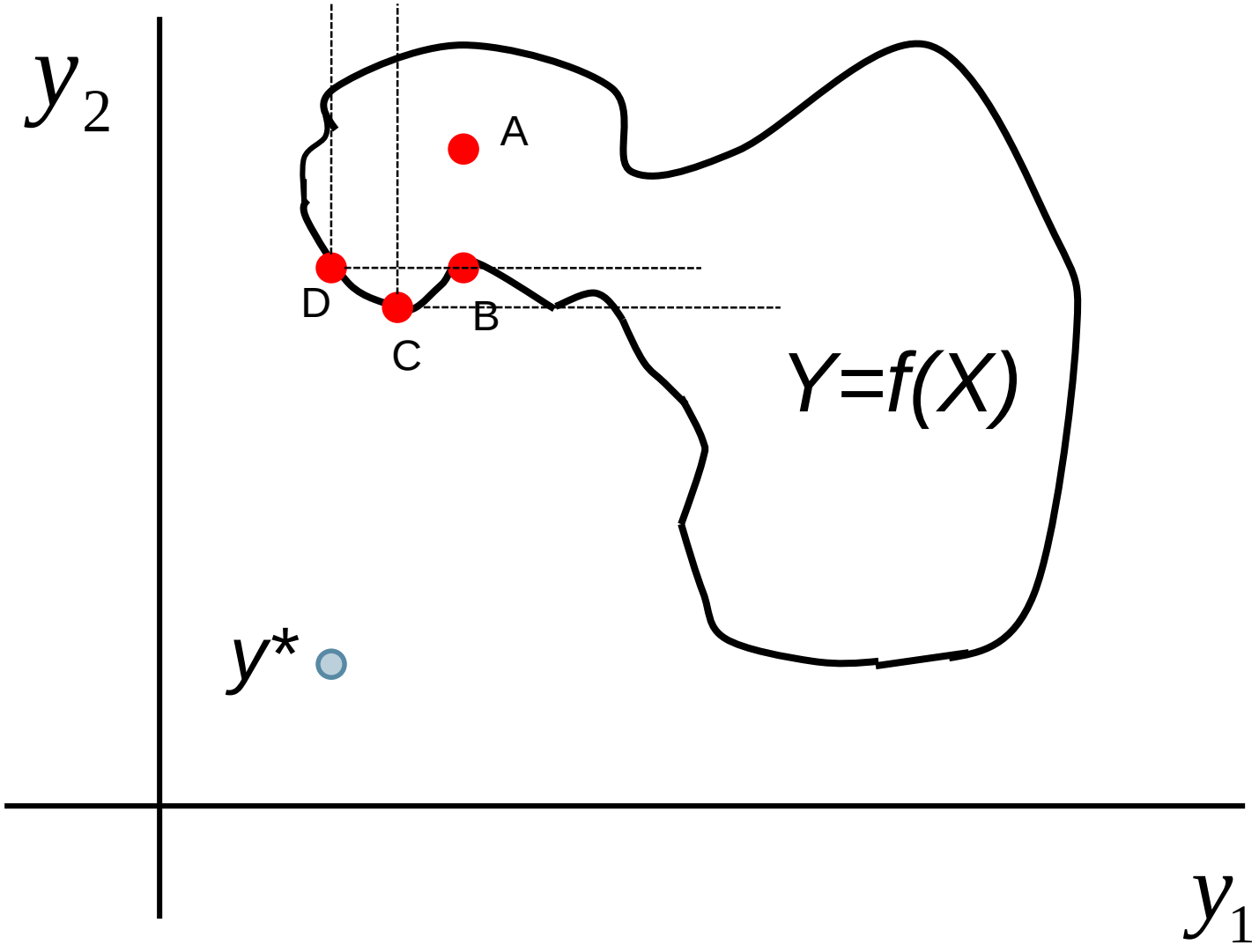
Feasible set in criterion space



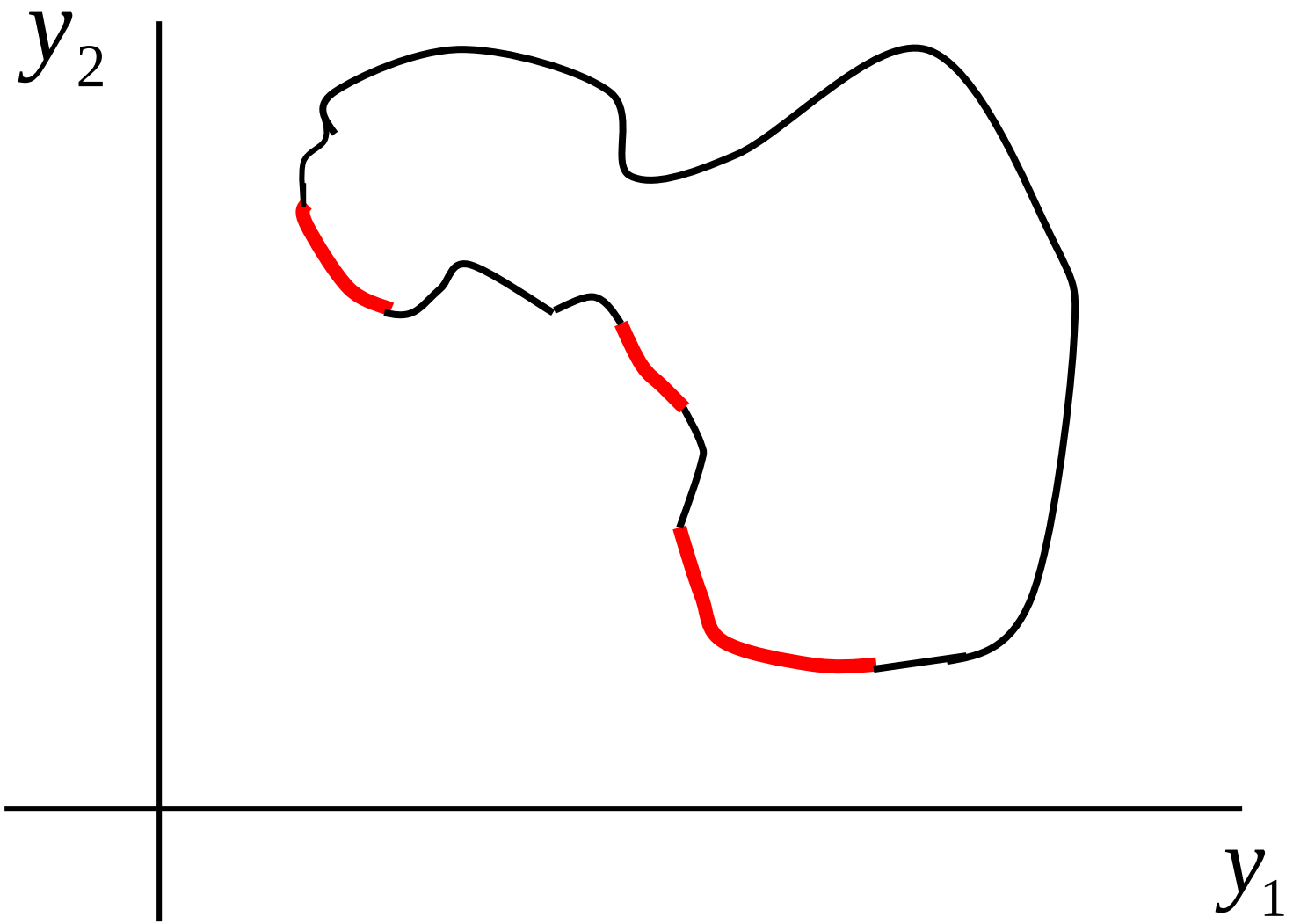
Pareto domination (minimization case)



Pareto domination (minimization case)

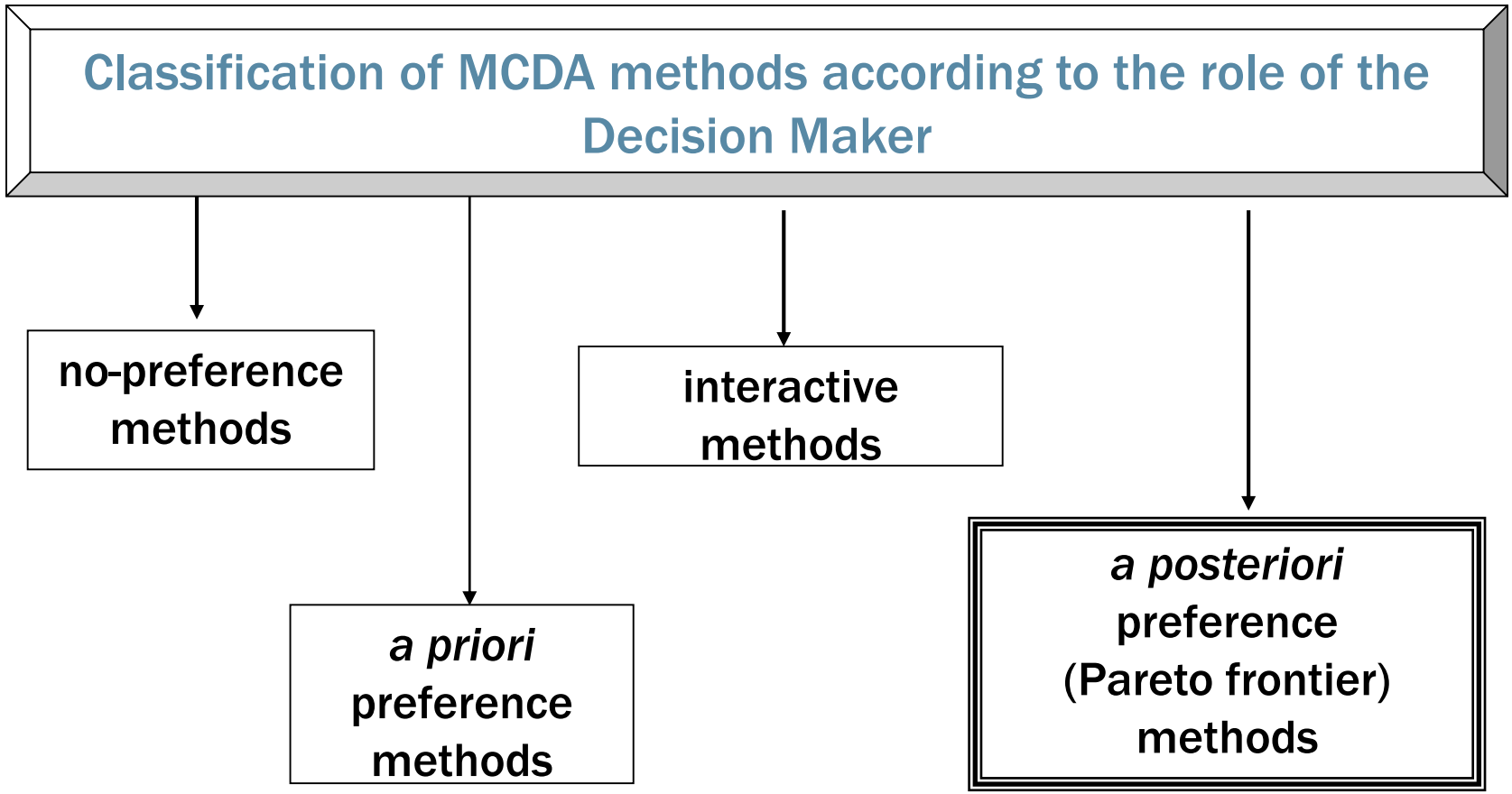


Non-dominated (efficient, Pareto) frontier



Decision maker (DM) is needed to select a unique solution from the set of Pareto-optimal solutions !

Classification of MCDA methods according to the role of the Decision Maker



✓ Pareto frontier methods

- In contrast to preference-oriented methods, they do not require complicated and time-consuming interactions with DM to identify preferences

- In contrast to *a priori* preference methods such as Goal Programming , they do not require that the DM specifies a goal without knowing information about its feasibility

✓ Key challenges to develop a Pareto frontier method

- ✓How to approximate the Pareto frontier

- ✓ How to provide stakeholders information about the Pareto frontier e. g.
  - ✓ By providing a list of the criterion points that belong to the Pareto frontier?
  - ✓ By providing a visualization of the Pareto frontier?

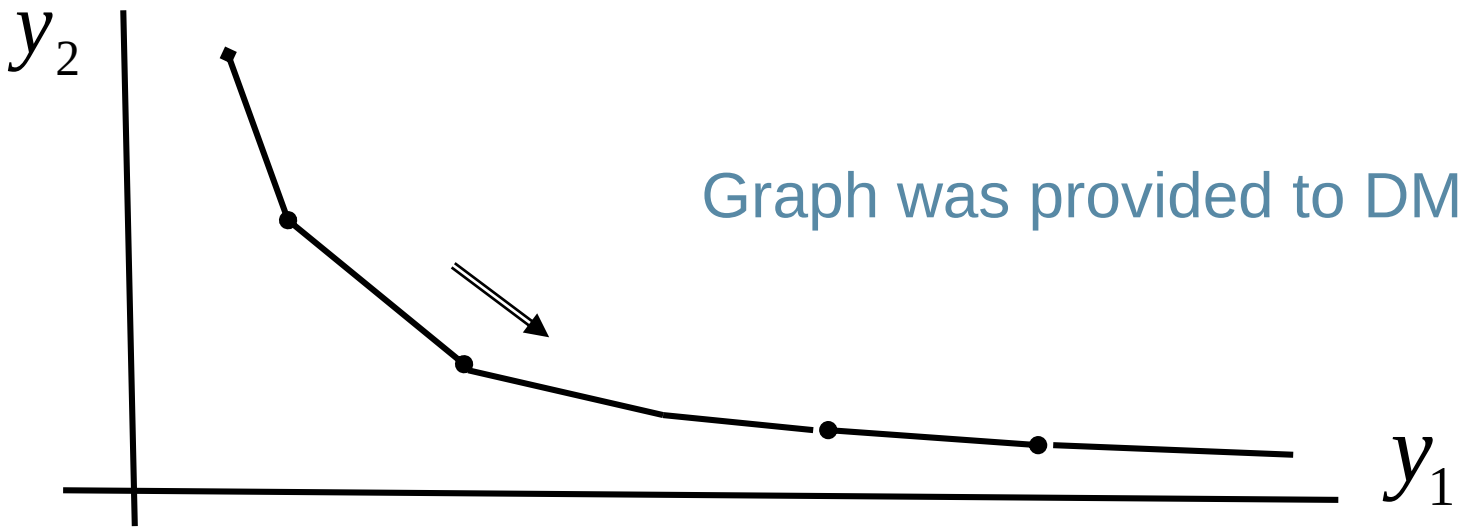
The first Pareto frontier method in MCO:

generating the Pareto frontier in linear bi-criterion problem (S.Gass and T.Saaty, 1955).

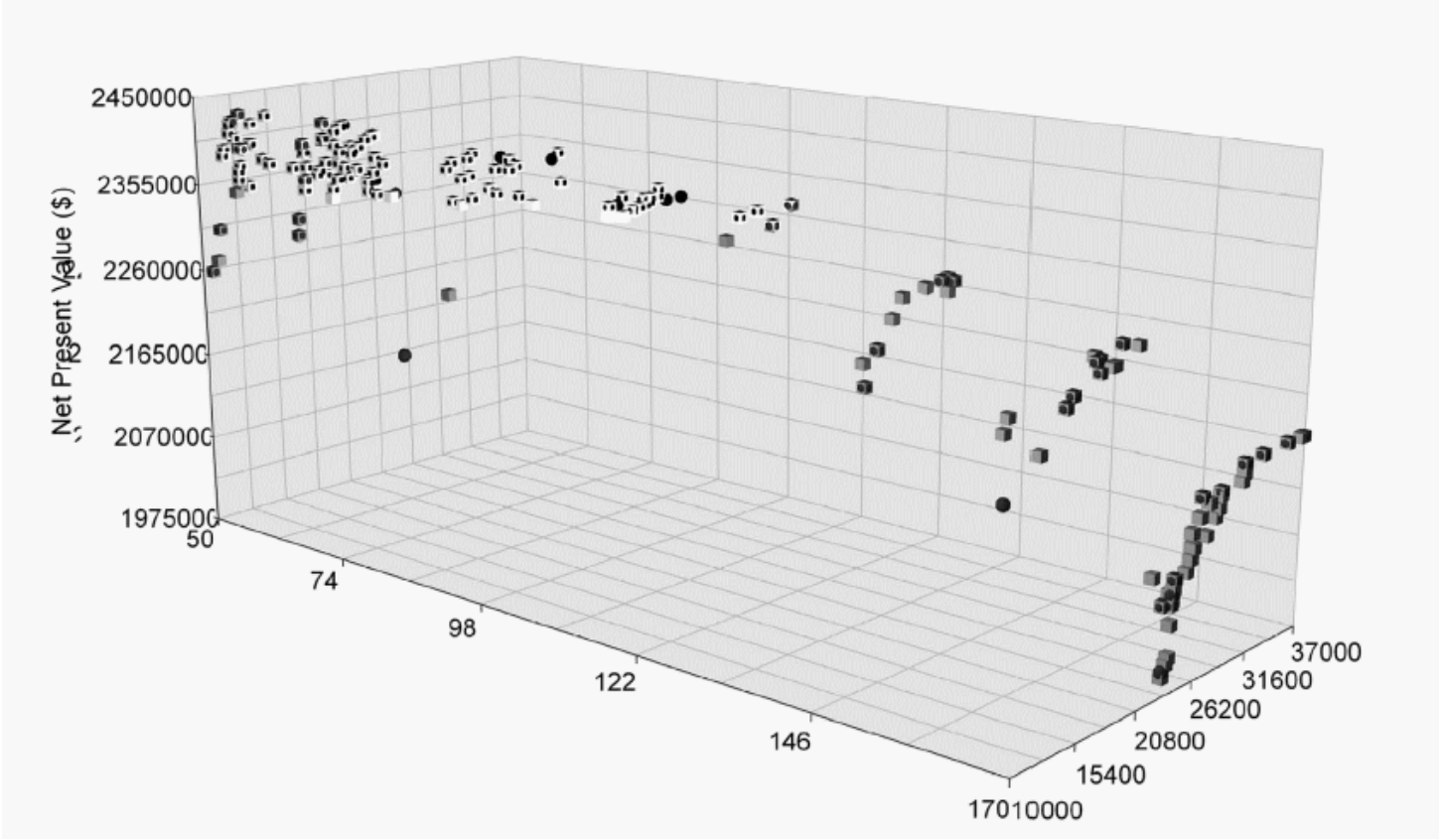
Parametric LP methods may be used for solving

$$\lambda y_1 + (1-\lambda) y_2 \to \min \quad z = Cx, \; Ax \le b$$

where λ changes from 0 to 1.



Graph was provided to DM!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2070000
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170
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26200
31600
37000

Thus, the question is:

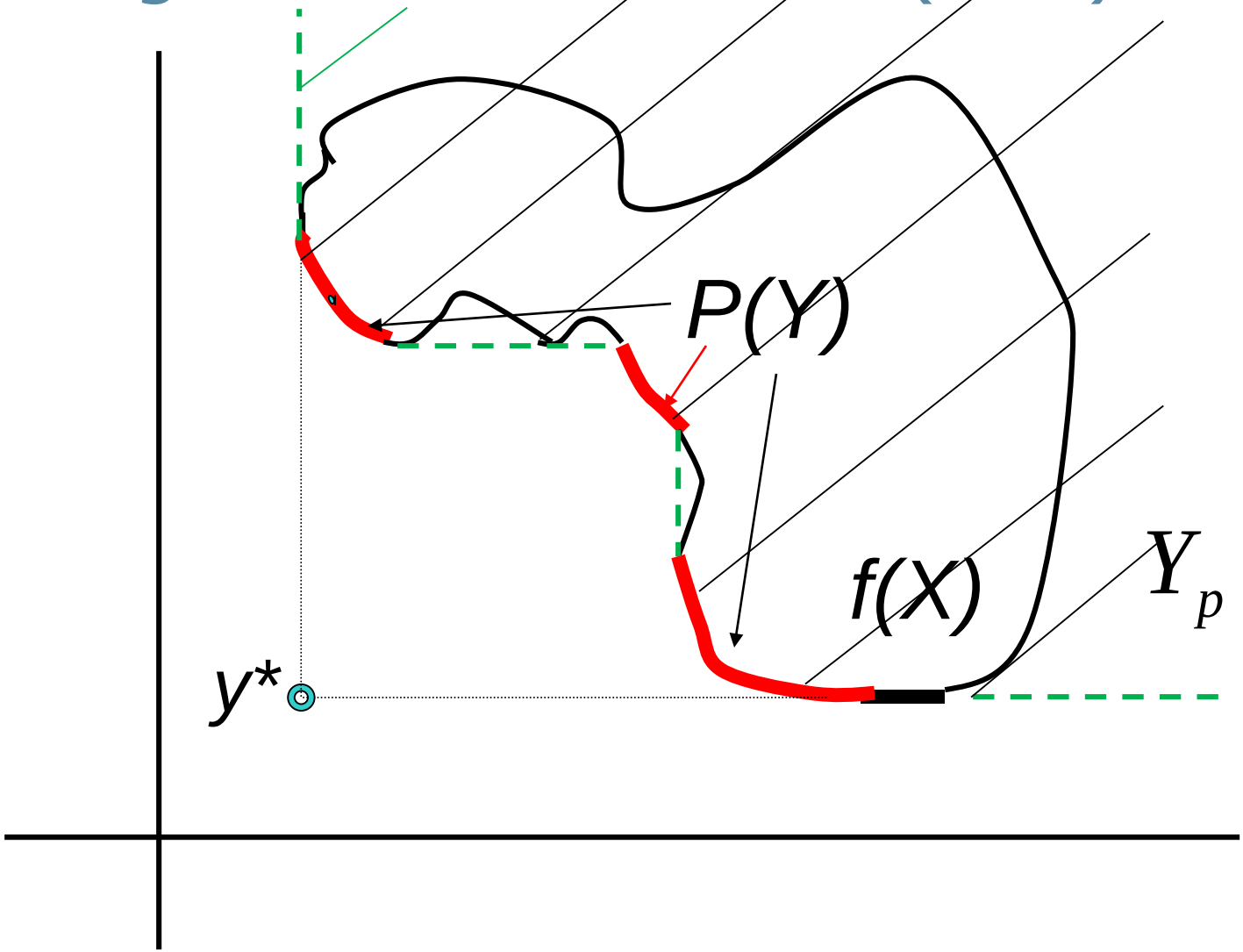
Is it possible and is it profitable to visualize the Pareto frontier in the case of more than two-three criteria?

Interactive Decision Maps technique answers:

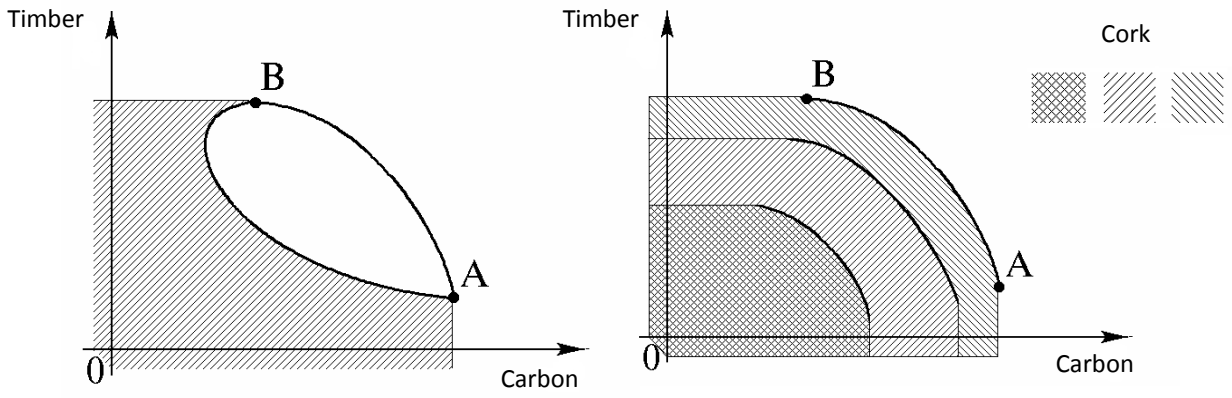
Yes, it is possible and profitable

Lotov, A.V., Bushenkov, V.A., Kamenev, G.K., 2004. Interactive Decision Maps. Approximation and Visualization of Pareto Frontier. In series: Applied Optimization, vol. 89. Kluwer Academic Publishers, New York.

Edgeworth-Pareto hull (EPH)



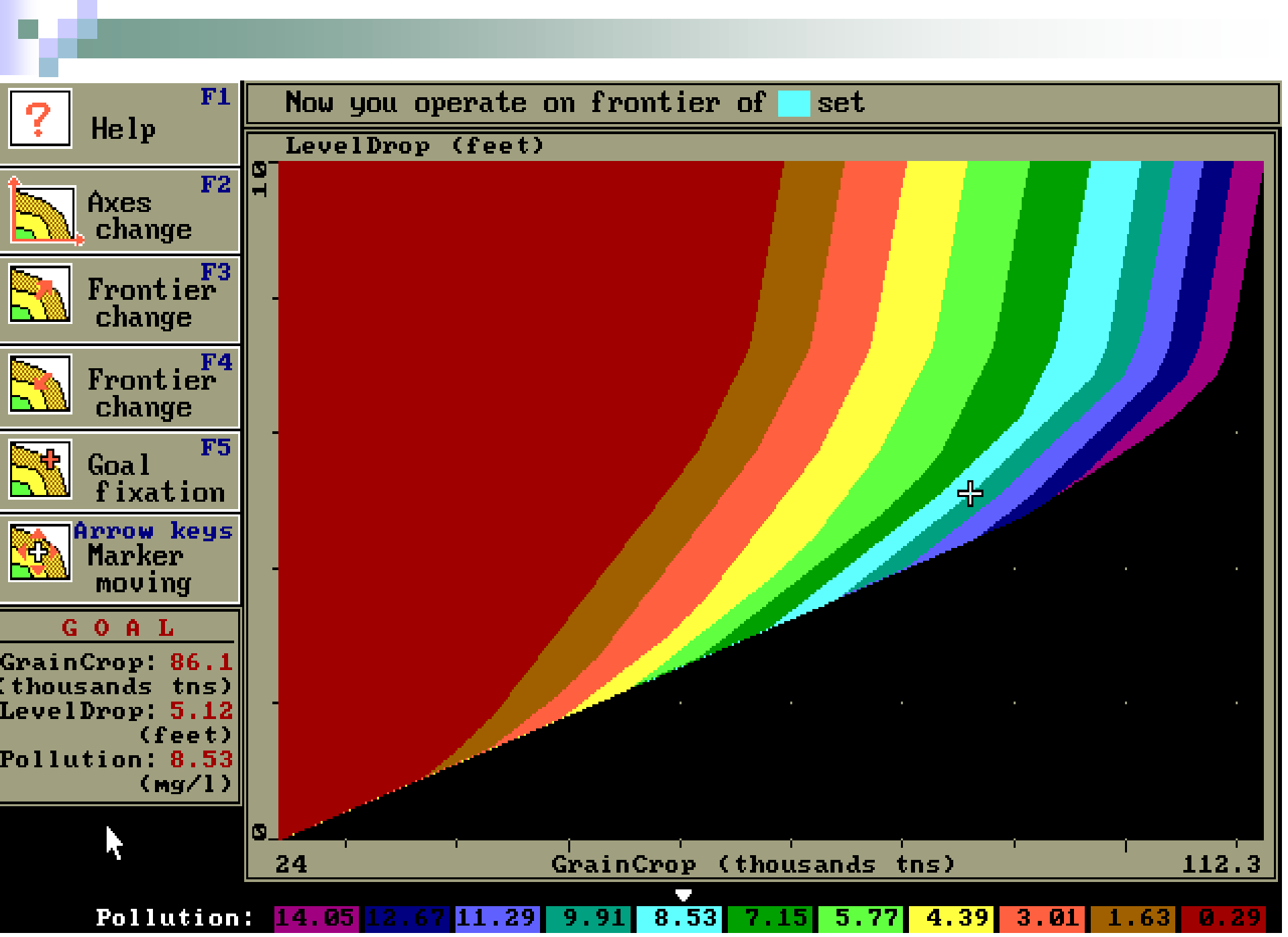
Interactive Decision Maps



Three dimensional decision map

Decision map is a collection of bi-objective slices of the Edgeworth-Pareto hull in the case of three criteria.

The decision maps are **used interactively**, they can be animated, zoomed, re-arranged, etc.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F2
Axes change
F3
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F4
Frontier change
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Goal fixation
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Now you operate on frontier of set
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GrainCrop (thousands tns)
112.3
G O A L
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LevelDrop: 5.12
(feet)
Pollution: 8.53
(mg/l)
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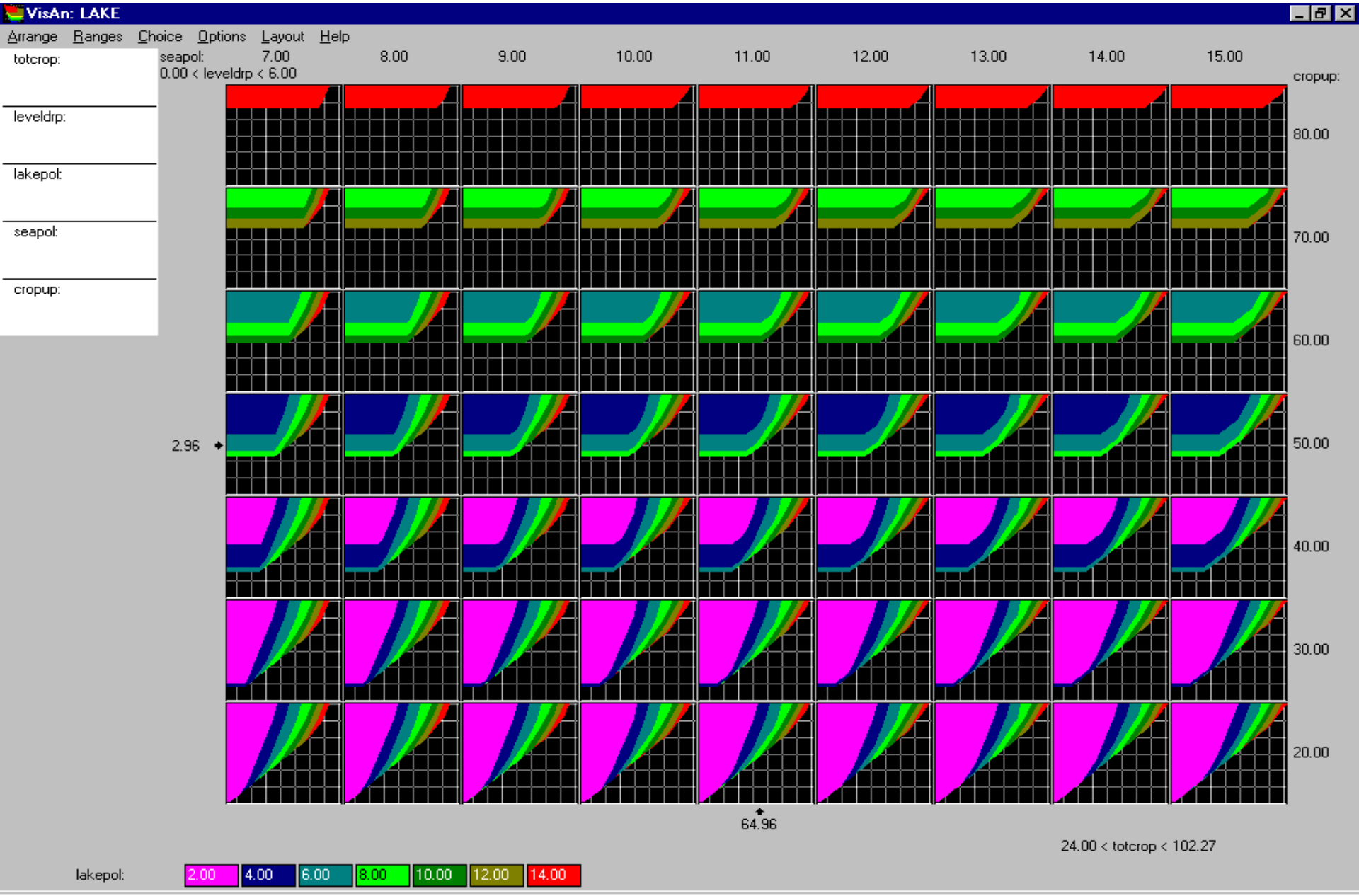
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totcrop:
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lakepol:
seapol:
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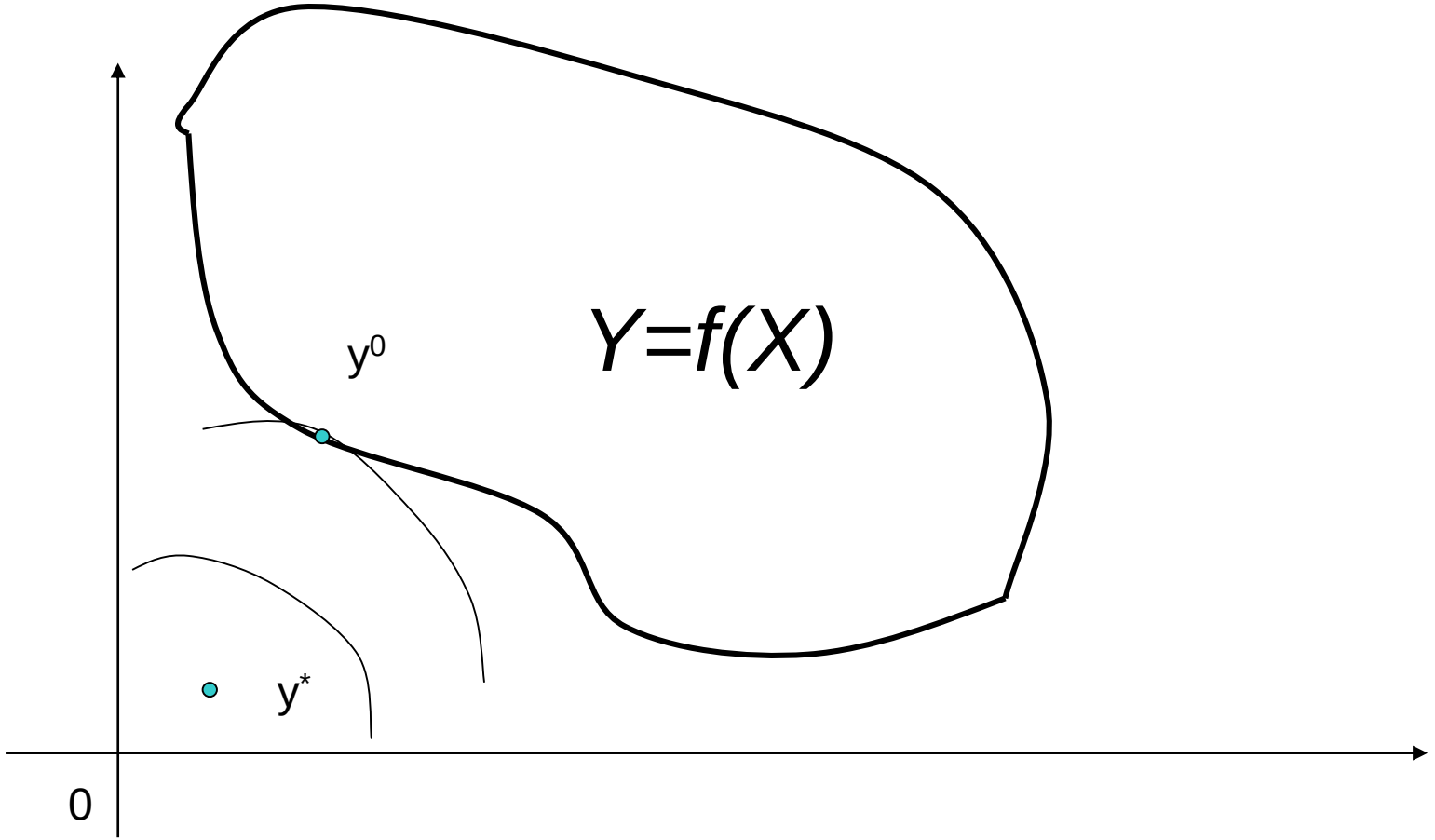
Feasible Goal Method / Interactive Decision Maps (FGM/IDM)

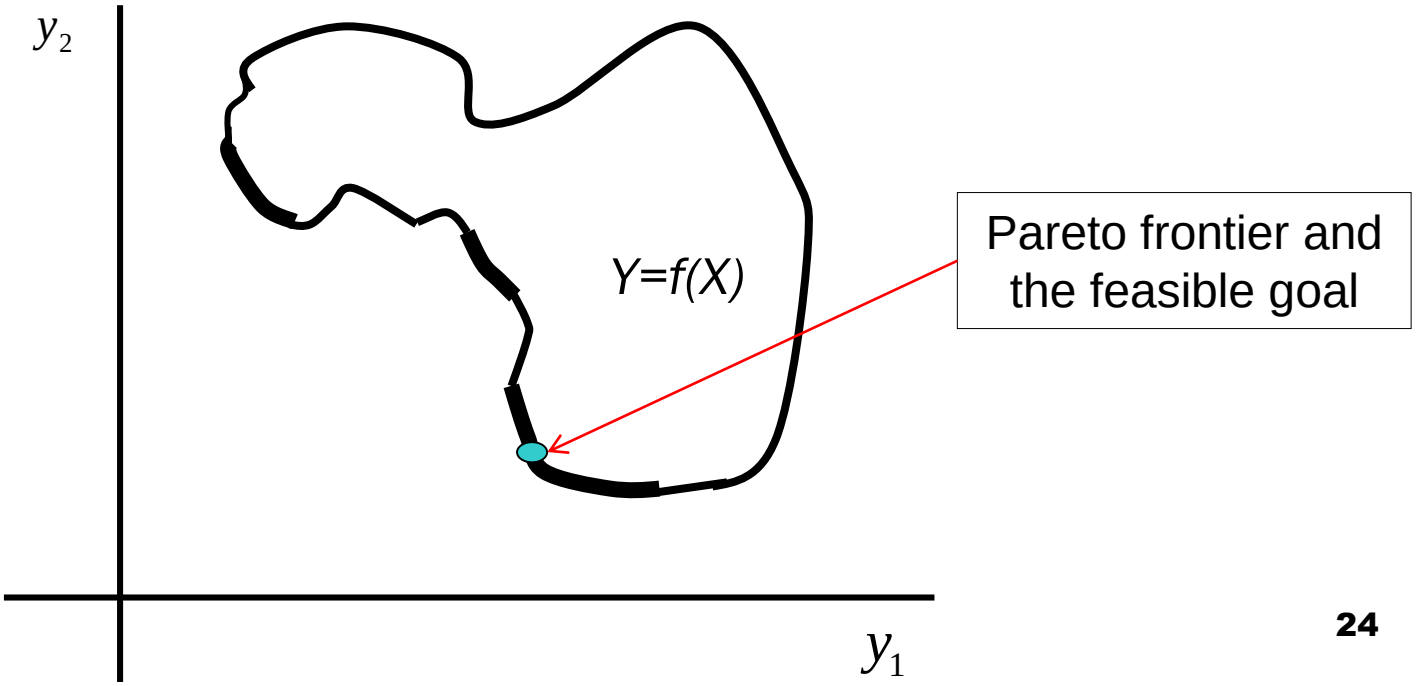
- ✓ Rather than forcing the DM to identify the goal (without information on the set Y=f(X))…

Feasible Goal Method / Interactive Decision Maps (FGM/IDM)

✓ … and then using some distance function to find the closest point of the set Y=f(X) …



Feasible Goal Method / Interactive Decision Maps (FGM/IDM)



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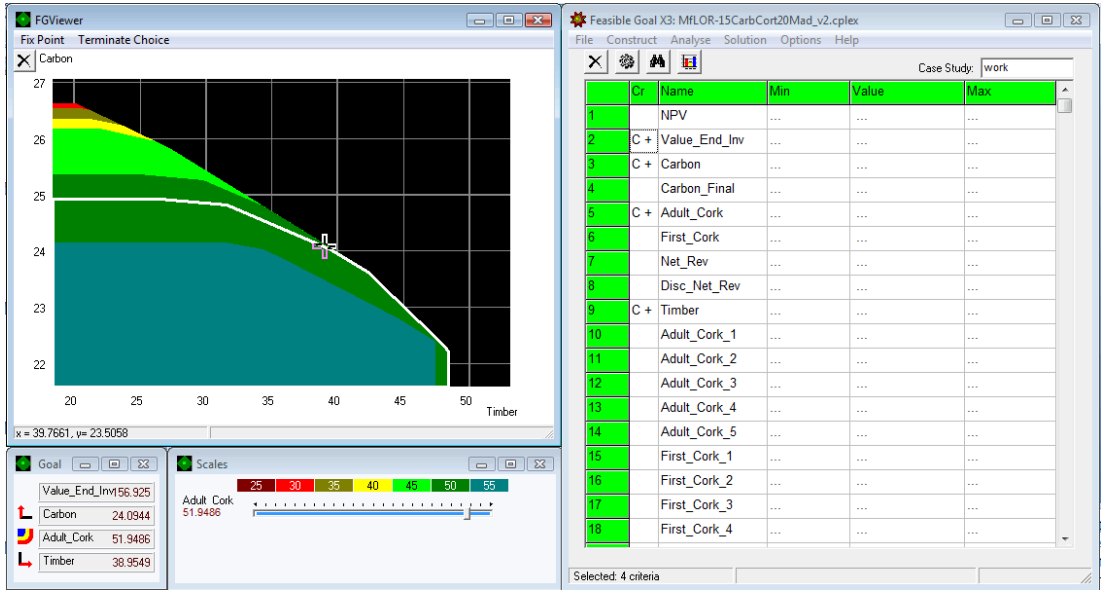
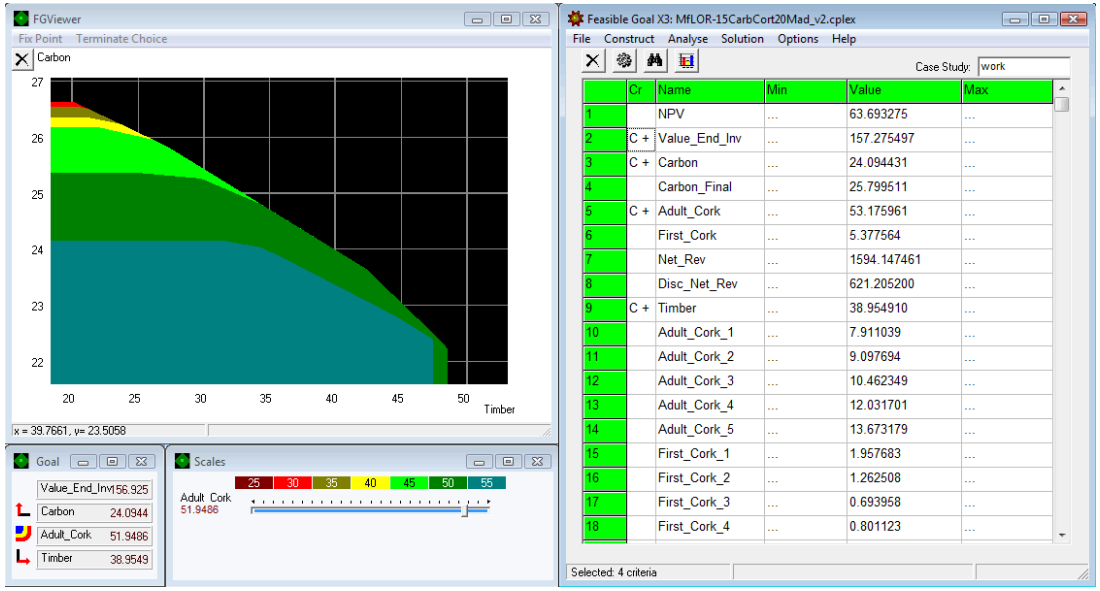


… the FGM technique provides criterion tradeoff information that is important for the DM to identify the preferable non-dominated feasible criterion point (goal).

It can be done **directly** at the non-dominated frontier by using the computer mouse.

Feasible Goal Method / Interactive Decision Maps (FGM/IDM)



	Cr	Name	Min	Value	Max
1		NPV	...	63.693275	...
2	C +	Value_End_Inv	...	157.275497	...
3	C +	Carbon	...	24.094431	...
4		Carbon_Final	...	25.799511	...
5	C +	Adult_Cork	...	53.175961	...
6		First_Cork	...	5.377564	...
7		Net_Rev	...	1594.147461	...
8		Disc_Net_Rev	...	621.205200	...
9	C +	Timber	...	38.954910	...
10		Adult_Cork_1	...	7.911039	...
11		Adult_Cork_2	...	9.097694	...
12		Adult_Cork_3	...	10.462349	...
13		Adult_Cork_4	...	12.031701	...
14		Adult_Cork_5	...	13.673179	...
15		First_Cork_1	...	1.957683	...
16		First_Cork_2	...	1.262508	...
17		First_Cork_3	...	0.693958	...
18		First_Cork_4	...	0.801123	...

✓ Reference Point Method (Wierzbicki, 1981):

$$\max_{i=1,2,..5}\left(\omega_i\left(y_i^*-y_i\right)\right) + \sum_{i=1}^{5}\varepsilon_i\left(y_i^*-y_i\right) \rightarrow \min$$

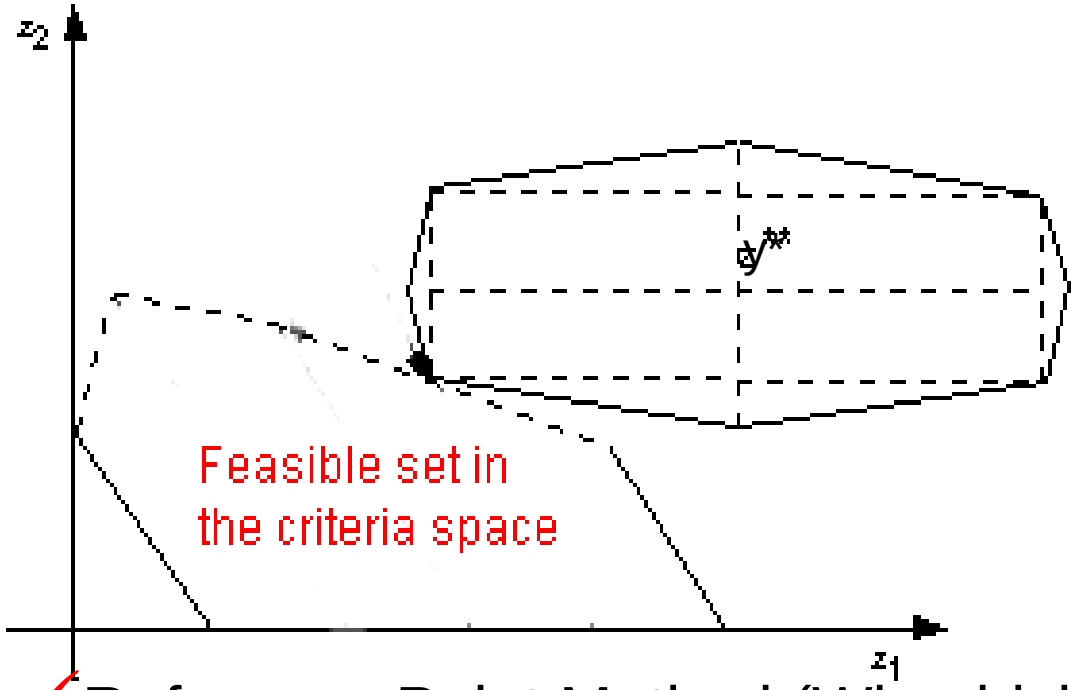
$$y = f(x), \quad x \in X$$

$y_i^*, \forall i$ - fixed in the IDM

$\omega_i \geq 0, \forall i$ - weights,

$\varepsilon_i \geq 0, \forall i$ - small constants

Feasible Goal Method / Interactive Decision Maps (FGM/IDM)



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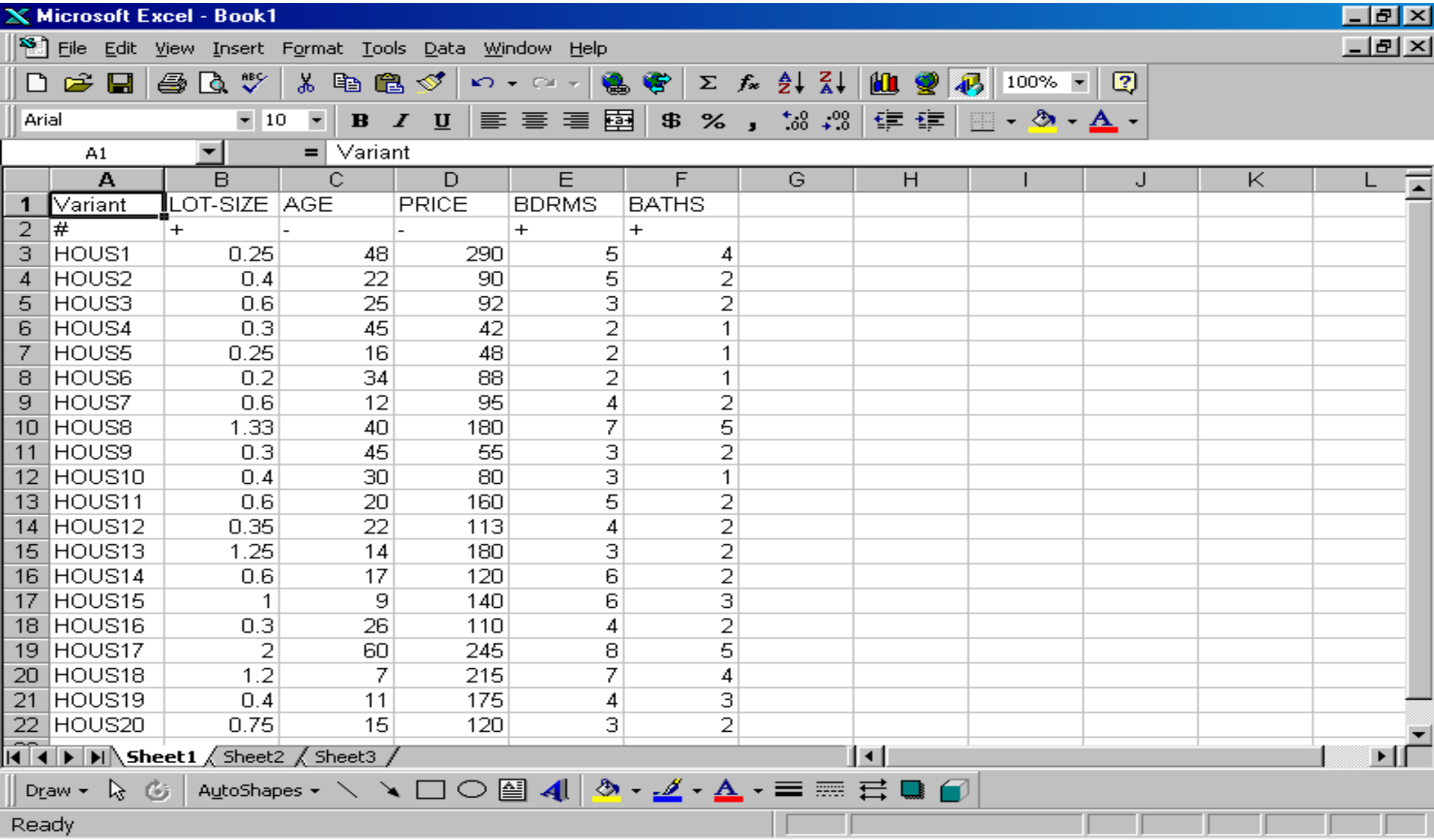
Reasonable Goals method / Interactive Decision maps (RGM / IDM) and its application in exploration of complex non-linear models

Reasonable Goal Method / Interactive Decision Maps (RGM/IDM)

A table of *N* decision alternatives (rows) is considered. Alternatives are given by a finite number (*m*) of attributes, which are used as the selection criteria (objectives)

Attribute	1	2	3	…	
Row 1	y_1^1	y_2^1	y_3^1		y_m^1
Row 2	y_1^2	y_2^2	y_3^2		y_m^2
Row 3	y_1^3	y_2^3	y_3^3		y_m^3
…					
…					
Row *N*	y_1^N	y_2^N	y_3^N		y_m^N

Example: Houses on sale



Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window Help

Arial 10 B I U

A1 = Variant

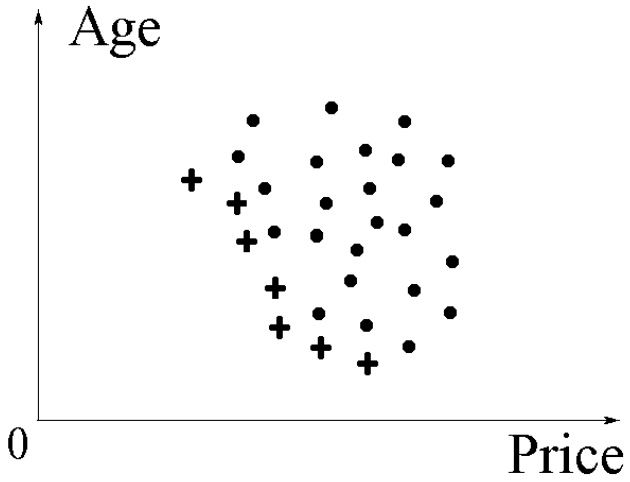
	A	B	C	D	E	F	G	H	I	J	K	L
1	Variant	LOT-SIZE	AGE	PRICE	BDRMS	BATHS						
2	#	+	-	-	+	+						
3	HOUS1	0.25	48	290	5	4						
4	HOUS2	0.4	22	90	5	2						
5	HOUS3	0.6	25	92	3	2						
6	HOUS4	0.3	45	42	2	1						
7	HOUS5	0.25	16	48	2	1						
8	HOUS6	0.2	34	88	2	1						
9	HOUS7	0.6	12	95	4	2						
10	HOUS8	1.33	40	180	7	5						
11	HOUS9	0.3	45	55	3	2						
12	HOUS10	0.4	30	80	3	1						
13	HOUS11	0.6	20	160	5	2						
14	HOUS12	0.35	22	113	4	2						
15	HOUS13	1.25	14	180	3	2						
16	HOUS14	0.6	17	120	6	2						
17	HOUS15	1	9	140	6	3						
18	HOUS16	0.3	26	110	4	2						
19	HOUS17	2	60	245	8	5						
20	HOUS18	1.2	7	215	7	4						
21	HOUS19	0.4	11	175	4	3						
22	HOUS20	0.75	15	120	3	2						

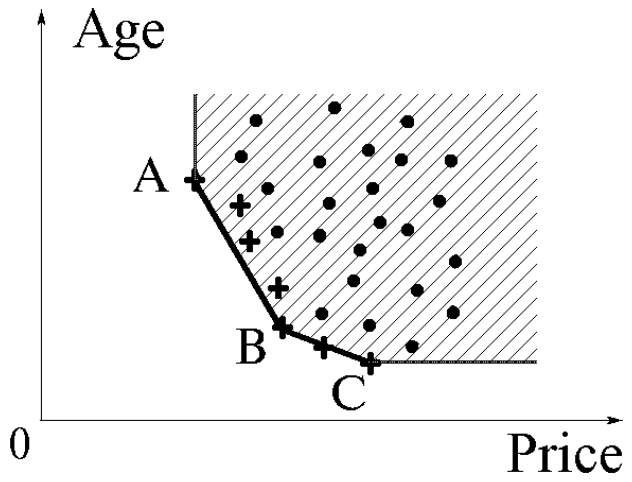
Sheet1 Sheet2 Sheet3

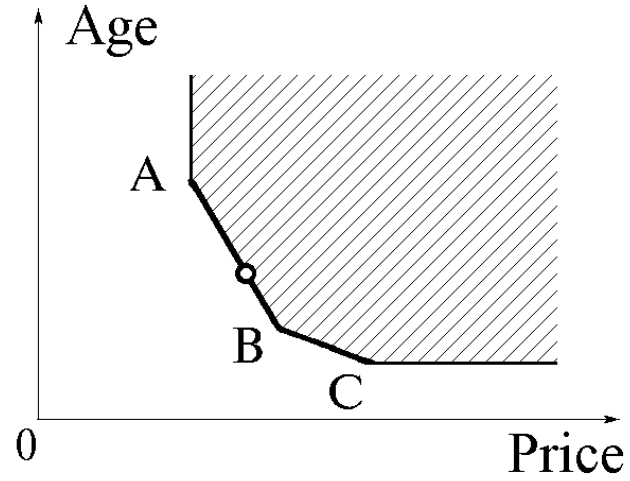
Draw AutoShapes

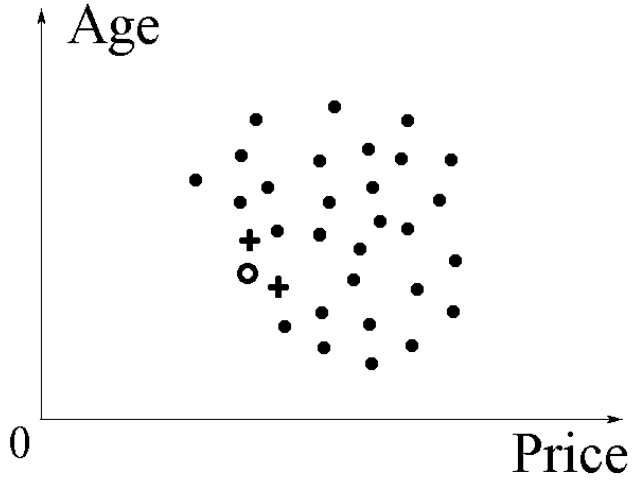
Ready

Reasonable Goal Method / Interactive Decision Maps (RGM/IDM)

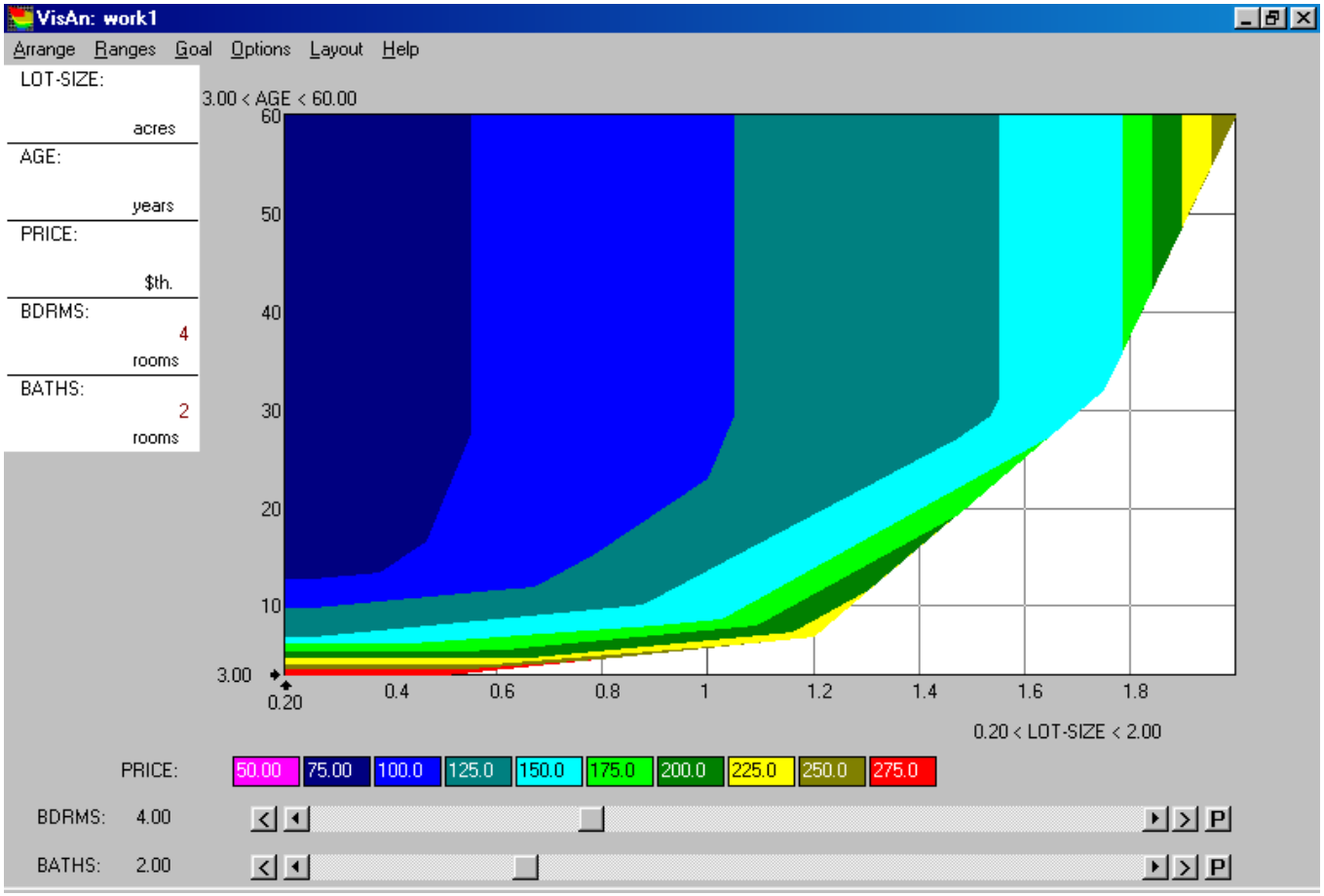








Reasonable Goal Method / Interactive Decision Maps (RGM/IDM)



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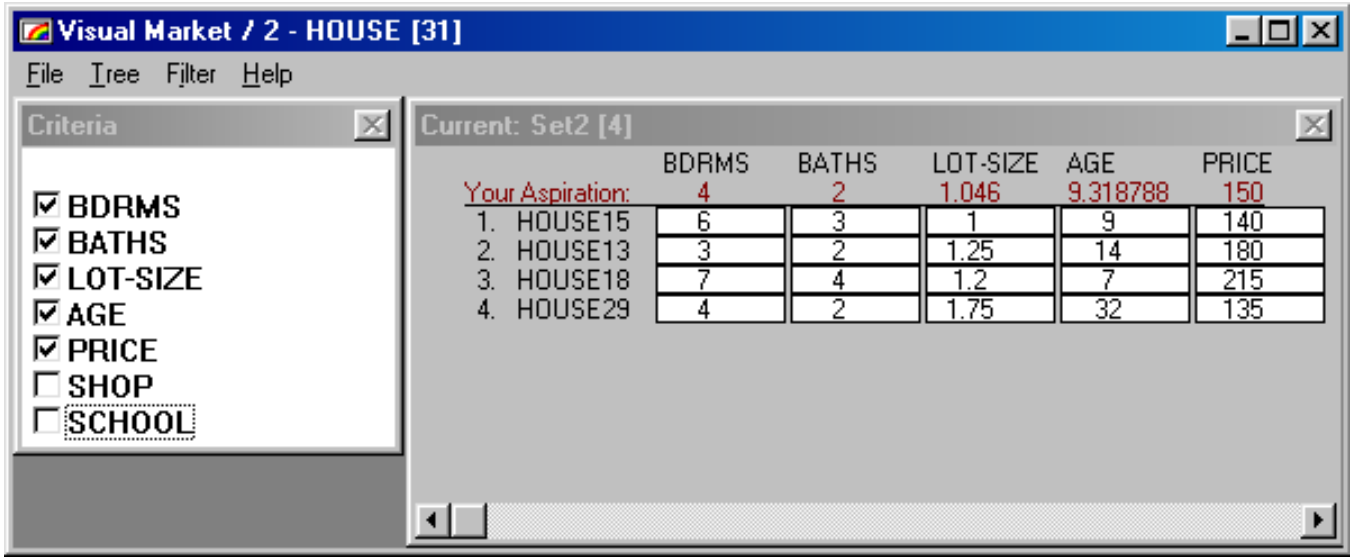
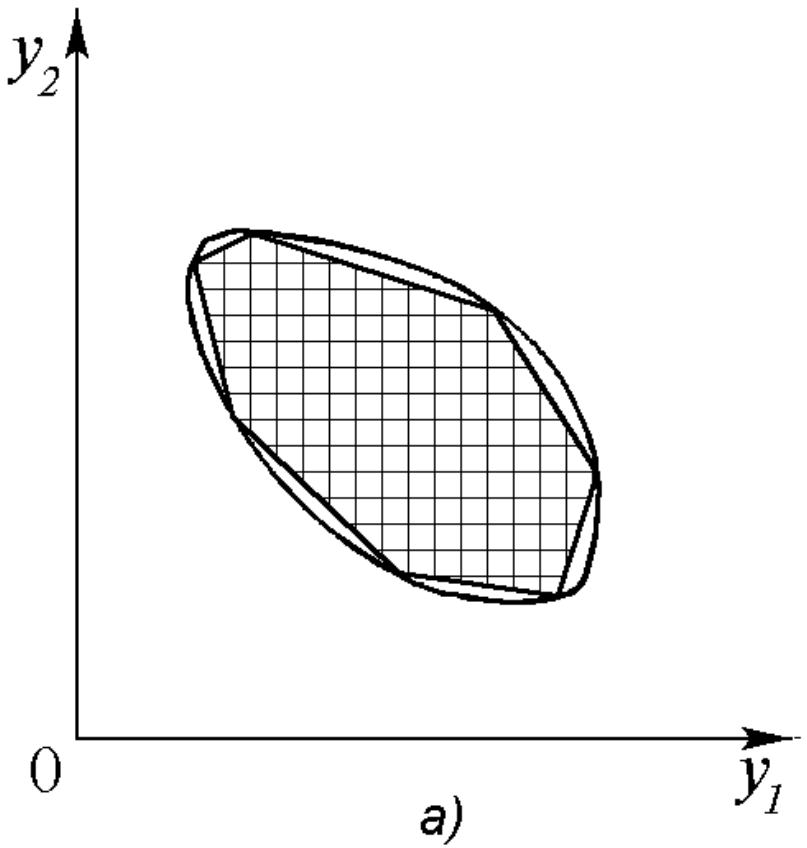
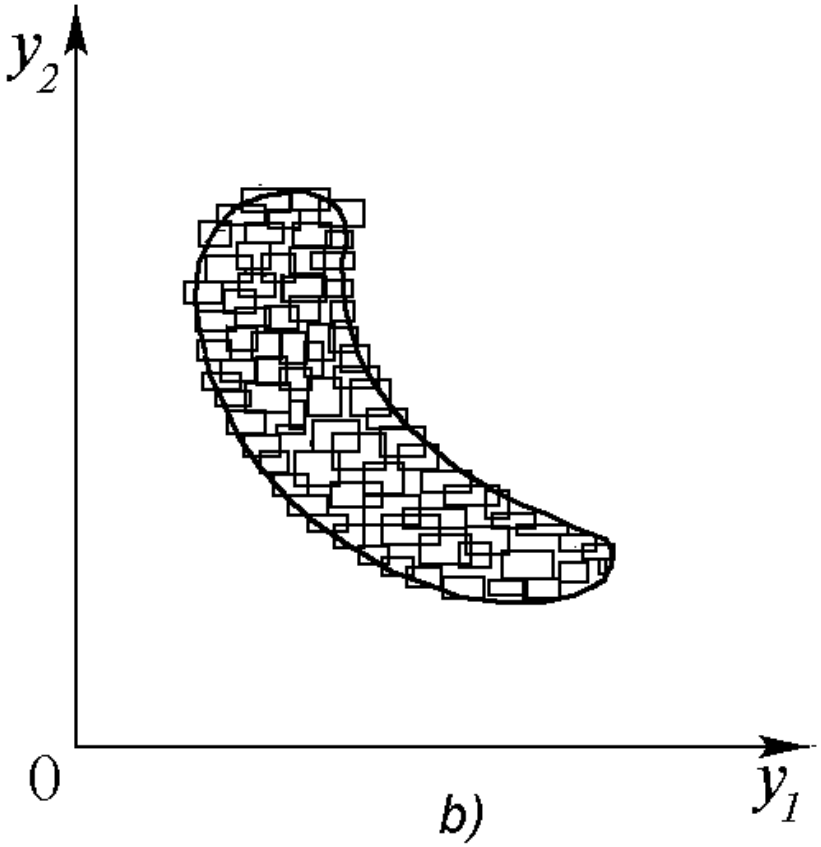


Thank You!



a)



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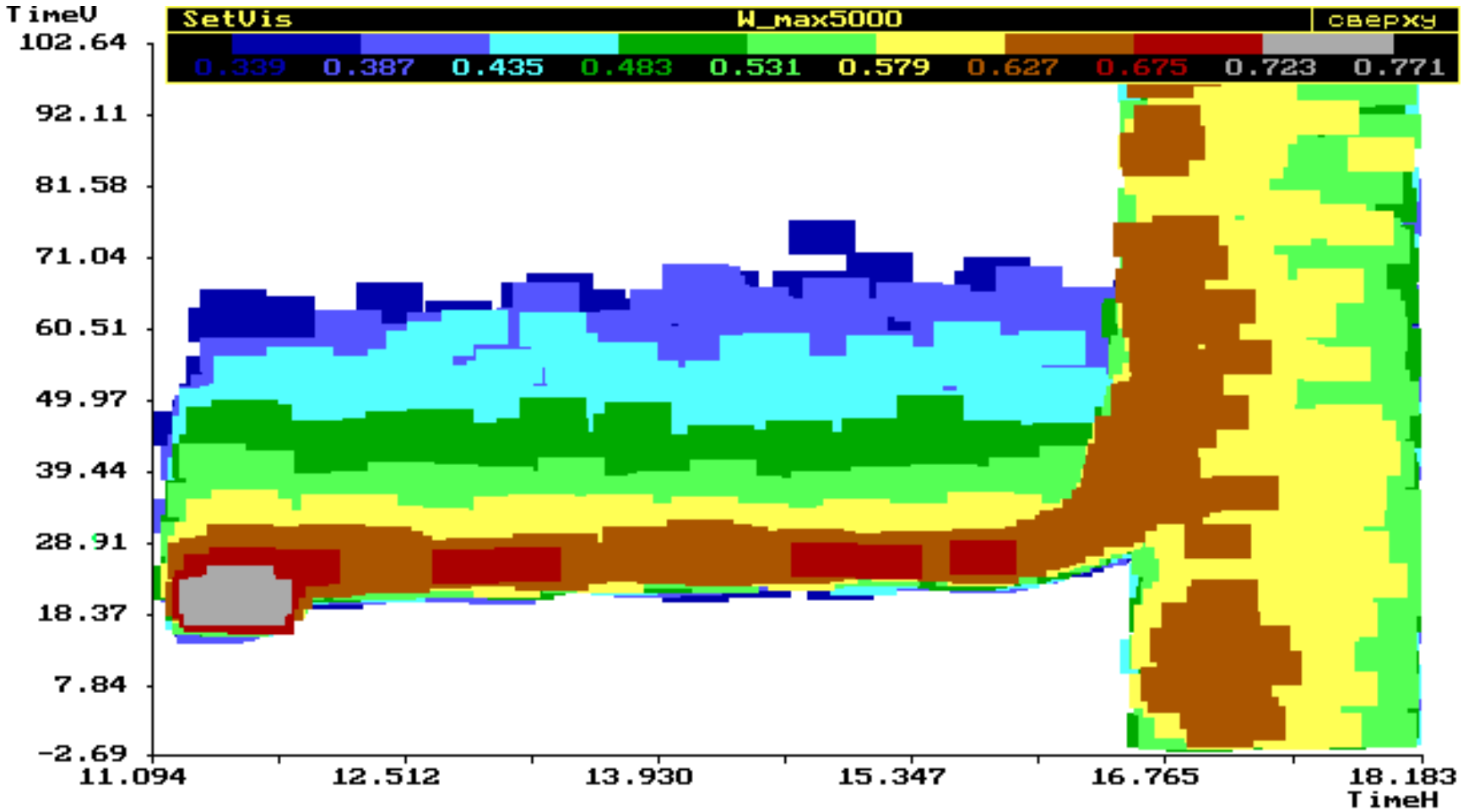
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areto Front Viewer - [Visualization]

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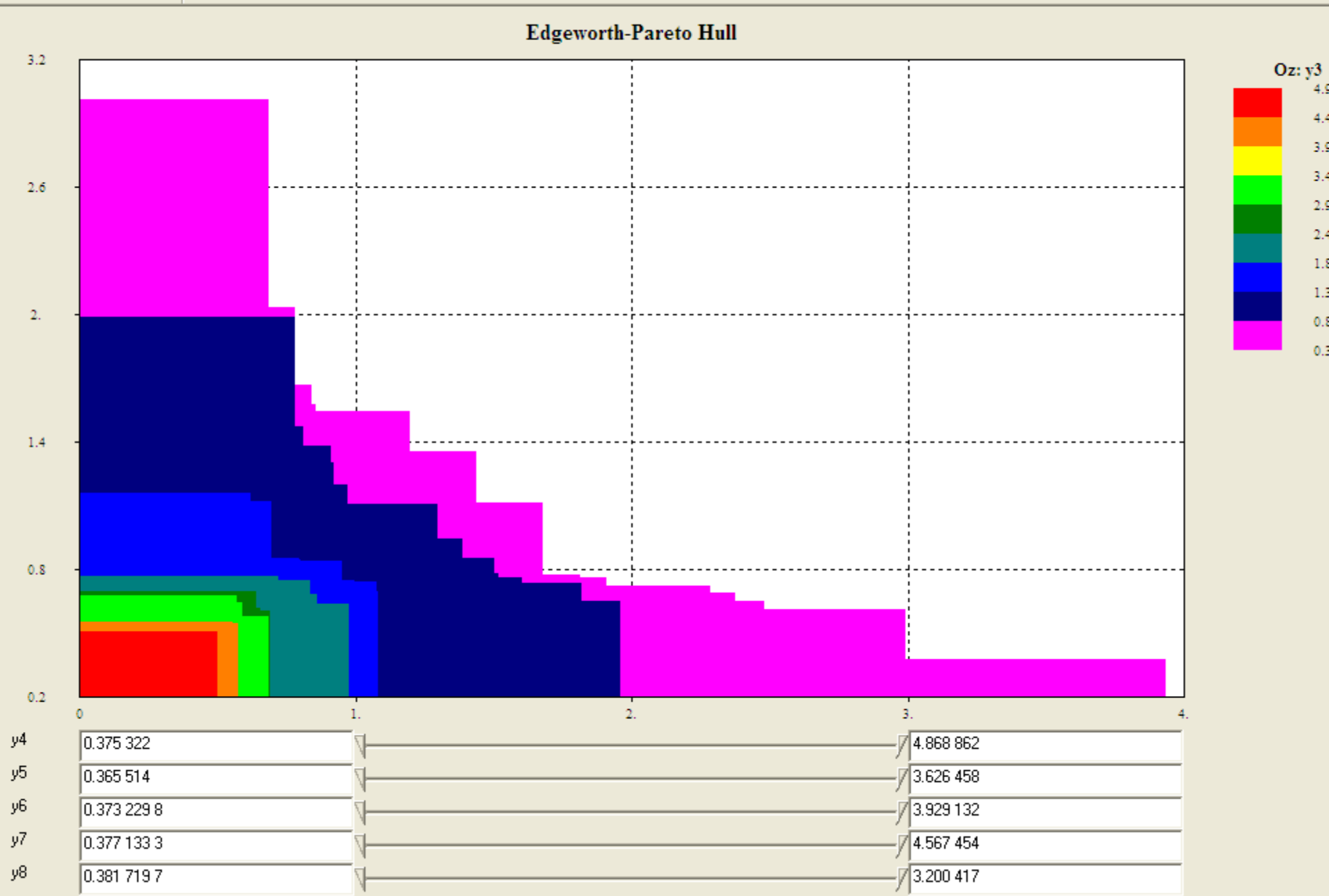

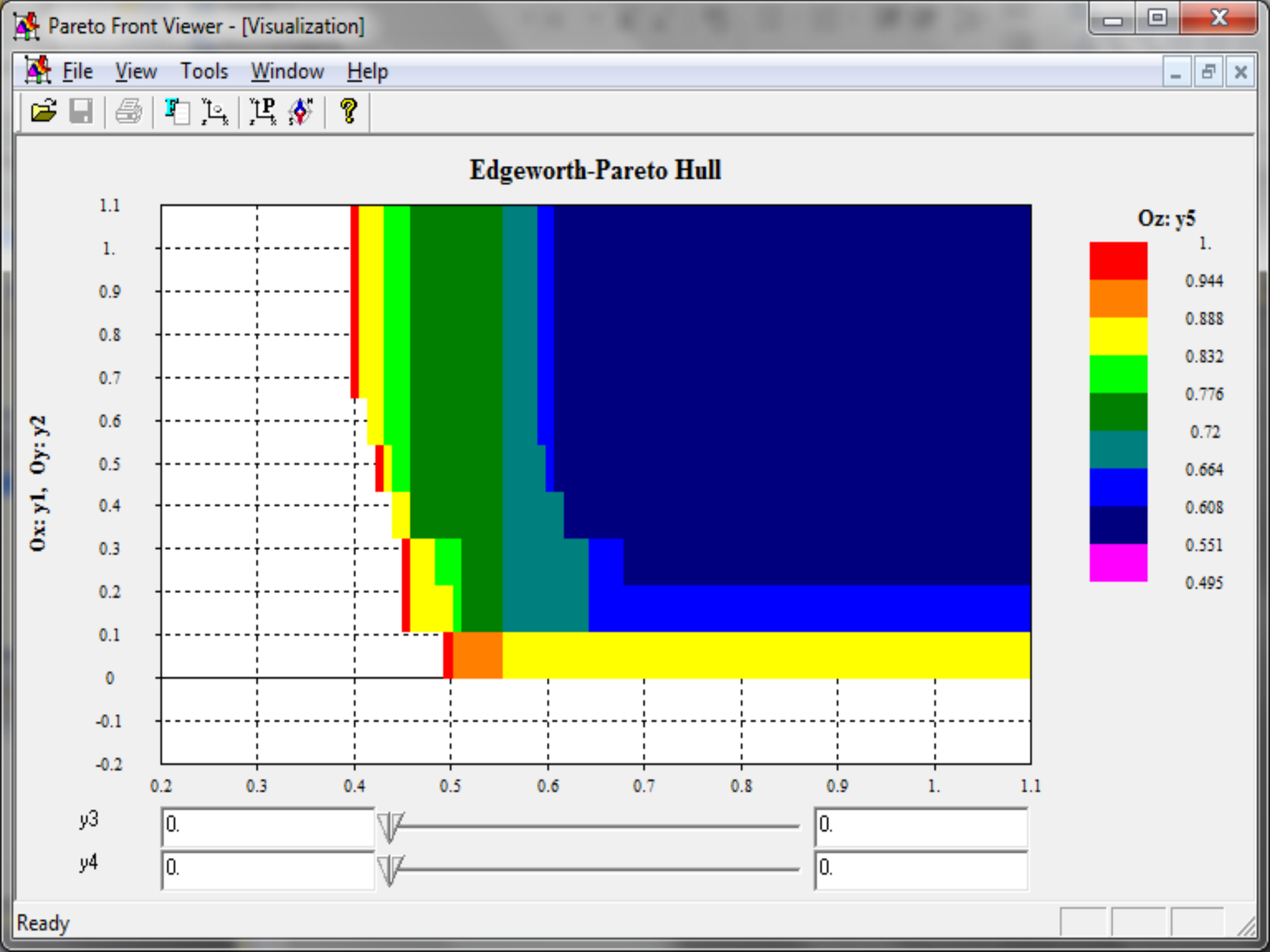
Pareto Front Viewer - [Visualization]
File
View
Tools
Window
Help
Edgeworth-Pareto Hull
Ox: y1, Oy: y2
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