UNIVERSITÀ DEGLI STUDI DI BRESCIA

A Model for Optimal Crop Selection Based on Conditional Value-at-risk

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Outline

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- Expected value approach
  - A first MILP model
- Conditional Value-at-Risk approach
  - A second MILP model
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- Conclusions and future work

Location

Rural area in Northern Italy

Crops

Outline

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CVaR
CSP(β)

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Rural area in Northern Italy

- Each **crop** requires a fixed sequence of **operations**
  - ploughing, seeding, etc.
- Each operation requires a specific **tool type**
  - ploughing requires a plough
  - working speed and/or operation cost may vary inside the same type and among crops
- Tools are mounted on a **tractor machine**
  - identical machines
  - limited number available

Problem definition

Second part

- Every operation of every crop has a **time window**
- **In a given time slot different operations for different crops can be performed simultaneously**, provided that:
  - the required tools and tractors are available
  - the time slot belongs to the appropriate time window
- **General task:** Optimal selection of crops and optimal assignment over time of their operations so to meet time windows and resource constraints, maximizing the expected profit

Literature review

- *Surveys on models to support cropping plan*
  - (Glen, 1987): mathematical models in farm planning
  - (Dury et al., 2012): models to support cropping plan and crop rotation

- *Papers most related to our work*
  - (Maruyama, 1972): stochastic LP for yield and price uncertainty
  - (Danok, McCarl, White, 1980): MILP for machinery selection and crop planning
  - (Annets and Audsley, 2002): MOLP for farm planning

- m number of crops
- n number of tool types
- A crop i is characterized by the ordered sequence of q_i operations:

$$j[i,1], j[i,2], \ldots, j[i,q_i],$$

where $j[i,k]$ is the index of the tool type needed to perform the k-th operation of crop i

- A binary index vector:

$$a_{i,j,k} = \begin{cases} 1, & \text{if } j \text{ is the } k\text{-th operation in crop } i; \\ 0, & \text{otherwise.} \end{cases}$$

EV-MILP model
Parameters

- $[s_{i,k}, f_{i,k}]$ time window for the k-th operation of crop i
- $[0, T] = [\min_i\{s_{i,1}\}, \max_i\{f_{i,q_i}\}]$ time horizon
- $\bar{r}_i$ expected revenue for one hectare cultivated with crop i
  - Obtained from historical data on prices and yields
- $h_{i,k,\ell}$ number of hectares that can be worked out in a time unit performing the k-th operation on crop i, using the ℓ-th tool of type $j[i,k]$
- $c_{i,k,\ell}$ time unit cost of using the ℓ-th tool of type $j[i,k]$ on the k-th operation of crop i
- w number of (identical) tractor machines
- H total number of hectares available for cultivation
- u_j number of tools of type j

- $I(j,t)$ subset of crop indices requiring a tool of type j that may be active at time t

$$I(j,t) = \{i : j = j[i,k], t \in [s_{i,k}, f_{i,k}] \text{ for some } k\}$$

- **Binary variables**:

$$y_{i,k,\ell,t} = \begin{cases} 1, & \text{if compatible machine } \ell \text{ is assigned} \\ & \text{to the } k\text{-th operation of crop } i \text{ at time } t; \\ 0, & \text{otherwise.} \end{cases}$$

- **Flow variables**:

$$z_{i,k,\ell} = \text{number of hectares where the } k\text{-th operation of crop } i \text{ is worked out using tool } \ell$$

- **Constraint 1**: budget constraint on the total area that can be farmed

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{i,j,1} \sum_{\ell=1}^{u_j} z_{i,1,\ell} \leq H$$

- **Constraint 2**: the hectares worked out by a given crop must be the same for every operation on that crop

$$\sum_{j=1}^{n} a_{i,j,k-1} \sum_{\ell=1}^{u_j} z_{i,k-1,\ell} - \sum_{j=1}^{n} a_{i,j,k} \sum_{\ell=1}^{u_j} z_{i,k,\ell} = 0$$

for all $i = 1, \ldots, m$ and $k = 2, \ldots, q_i$

- **Constraint 3**: at any time unit t every single tool can be assigned to at most one operation on some crop

$$\sum_{i \in I(j,t)} \sum_{k=1}^{q_i} a_{i,j,k}\, y_{i,k,\ell,t} \leq 1$$

for all $j = 1, \ldots, n$; $\ell = 1, \ldots, u_j$; $t \in [0, T]$

- **Constraint 4**: the number of tool-tractor pairs active at any time must not be greater than w

$$\sum_{j=1}^{n} \sum_{i \in I(j,t)} \sum_{\ell=1}^{u_j} \sum_{k=1}^{q_i} a_{i,j,k}\, y_{i,k,\ell,t} \leq w$$

for all $t \in [0, T]$

- **Constraints 5 and 6**: a necessary and sufficient quantity of resources must be allocated to every operation of every crop

$$\sum_{j=1}^{n} a_{i,j,k} h_{i,j,\ell} \left(\sum_{t=s_{i,k}}^{f_{i,k}} y_{i,k,\ell,t} - 1 \right) \leq z_{i,k,\ell} \leq \sum_{j=1}^{n} a_{i,j,k} h_{i,j,\ell} \sum_{t=s_{i,k}}^{f_{i,k}} y_{i,k,\ell,t}$$

for all $i = 1 \ldots, m$; $k = 1, \ldots, q_i$; $\ell = 1, \ldots, u_{j[i,k]}$

- **Constraint 7 and 8**: nonnegativity and binary restriction

$$z_{i,k,\ell} \geq 0 \text{ and } y_{i,k,\ell,t} \in \{0, 1\}$$

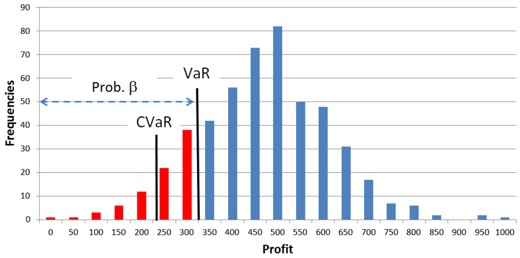
for all i, k, ℓ, and t

- **Objective**: difference between expected revenues and (certain) costs

$$\sum_{i=1}^{m}\left(\bar{r}_i \sum_{j=1}^{n} a_{i,j,1} \sum_{\ell=1}^{u_j} z_{i,1,\ell} - \sum_{k=1}^{q_i}\sum_{j=1}^{n} a_{i,j,k} \sum_{\ell=1}^{u_j} c_{i,j,\ell} \sum_{t=s_{i,k}}^{f_{i,k}} y_{i,k,\ell,t}\right)$$

- **CSP - Crop Selection Problem**
  - Maximise the above **Objective** subject to **Constraints 1-8**

- In model CSP, profit $p(\mathbf{x}, \mathbf{r})$ is a function of our decision ($\mathbf{x}$) and uncertain prices ($\mathbf{r}$)
- **Value-at-Risk** (VaR): $Prob\{p \leq VaR\} = \beta$ (for a fixed β)
- **Conditional Value-at-Risk** (CVaR):
  Conditional expectation of profits lower than VaR



- CVaR is a coherent risk measure with several nice properties (Pflug, 2000)

- For a given value of β, we pursue risk minimization through CVaR maximization
- Given S scenarios, this can be done (approximately) through the following model (Rockafellar and Uryasev, 2000):

$$\max \ \eta - \frac{1}{\beta}\sum_{s=1}^{S}\pi_s d_s$$

$$\text{subject to } \eta - p(\mathbf{x}, \mathbf{r}_s) \leq d_s \quad (s = 1, \cdots, S)$$

$$d_s \geq 0 \ (s = 1, \cdots, S), \mathbf{x} \in X$$

where:

- $\mathbf{r}_s$ is the s-th possible price realization (scenario); π_s is its probability
- X is the set of feasible crop assignments

- $r_{i,s}$ revenue of crop i under scenario s
- $\pi_s = 1/S$ for all scenarios
- **CSP(β):**

$$\max \quad \eta - \frac{1}{\beta S}\sum_{s=1}^{S} d_s$$

$$\text{subject to} \quad \eta - \sum_{i=1}^{m}\left(r_{i,s}\sum_{j=1}^{n} a_{i,j,1}\sum_{\ell=1}^{u_j} z_{i,1,\ell} - \sum_{k=1}^{q_i}\sum_{j=1}^{n} a_{i,j,k}\sum_{\ell=1}^{u_j} c_{i,j,\ell}\sum_{t=s_{i,k}}^{f_{i,k}} y_{i,k,\ell,t}\right) \le d_s \quad (s = 1,\dots,S)$$

$$d_s \ge 0 \quad (s = 1,\dots,S)$$

+ CSP constraints

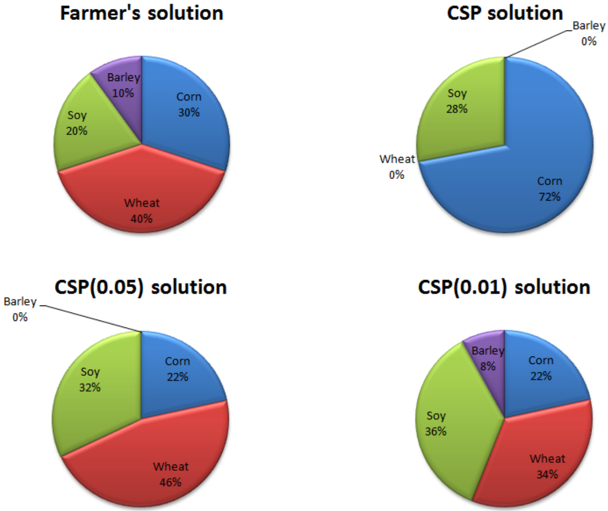
Data

- $H = 100$ hectares; $m = 4$ crops
- $w = 4$ tractors
- $n = 9$ ops./tool types; $|u_j| = 2$ for all j
- Operating costs and working speeds
  - Source: Farmer
- Prices: monthly prices from 2000 to 2009
  - Source: **ISMEA**
- Yield per hectare: yearly averages from 2000 to 2009
  - Source: **www.istat.it**
  - Each average yield perturbed 5 times according to a Beta distribution
- A total of $120 \times 5 = 600$ scenarios

Implementation

- Models implemented using IBM ILOG CPLEX Optimization Studio 12.5
  - CPLEX solver used the default settings
- Code run on an Intel Core i5 2.70GHz, 4MB RAM processor with 64 bit Windows 7 Pro
- Model size and Computational times
  - **CSP** model: 4092 rows and 809 columns
    - reduced to 349 rows and 618 columns
    - **solved to optimality in less than 1 sec. of total time**
  - **CSP(β)** with $\beta = 0.05, 0.01$: 4693 rows and 1410 columns
    - reduced to 950 rows and 1219 columns
    - **best integer solution found in the first 10 seconds and then stalling with a gap steadily around 0.10%**

Crop mix

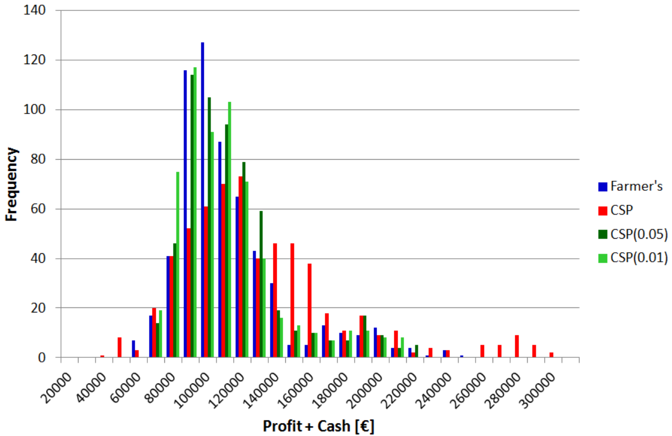

Farmer's solution
Barley 10%
Corn 30%
Soy 20%
Wheat 40%
CSP solution
Barley 0%
Soy 28%
Wheat 0%
Corn 72%
CSP(0.05) solution
Barley 0%
Soy 32%
Corn 22%
Wheat 46%
CSP(0.01) solution
Barley 8%
Corn 22%
Soy 36%
Wheat 34%

Initial budget: 15000 euro

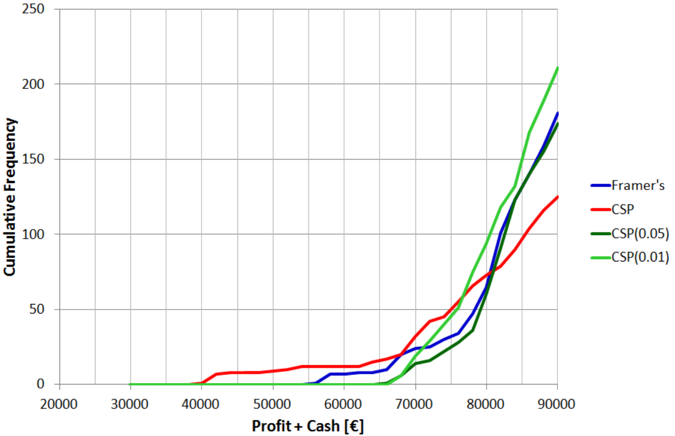
		CSP	
Statistics	**Farmer's sol.**	**Value**	**Variation**
Ave. Profit	101 124.87	124 628.50	23%
Cost	8090.00	13223.50	63%
Cash surplus	6910.00	1776.50	
Ave. Profit + Cash	108034.87	126405.00	17%
Max Profit	235 712.82	289 832.27	23%
Min Profit	48 559.39	37 149.23	-23%
Variance	1.0449E+09	2.2801E+09	118%
		CSP(0.05)	
Statistics		**Value**	**Variation**
Ave. Profit		99 921.02	-1%
Cost		6154.50	-24%
Cash surplus		8845.50	
Ave. Profit + Cash		108766.52	1%
Max Profit		209 998.52	-11%
Min Profit		56 723.22	17%
Variance		9.1787E+08	-12%
		CSP(0.01)	
Statistics		**Value**	**Variation**
Ave. Profit		97 076.49	-4%
Cost		6513.50	-19%
Cash surplus		8486.50	
Ave. Profit + Cash		105562.99	-2%
Max Profit		200 741.08	-15%
Min Profit		57 649.54	19%
Variance		8.6929E+08	-17%

Profit Distributions

Frequency



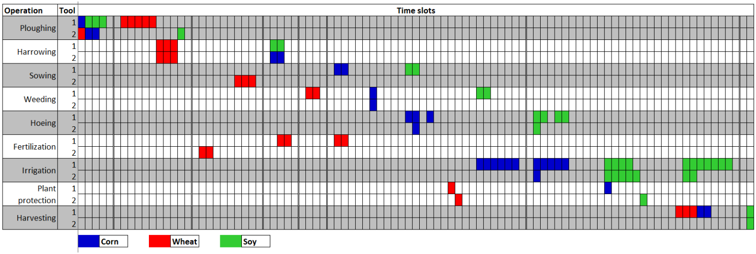
- CSP has a very large range of outcomes



- CSP(β) preserves from very bad outcomes

Gantt Chart

- CSP(0.05) solution (not all time slots displayed):



- Resources (tools, tractors) could be better exploited

Conclusions

- **Crop mix problem** modelled as a **CVaR maximization problem** with **scheduling constraints**
  - validated on a real case
  - more balanced solutions wrt an expected value maximization
- **Extensions** and future work:
  - evaluation of the effects of different resource configurations and the advantages of tool sharing
  - richer models, including decisions about the resources and their cost
  - deeper computational study