Analysing and Integrating Risk Attitudes in Forest Management

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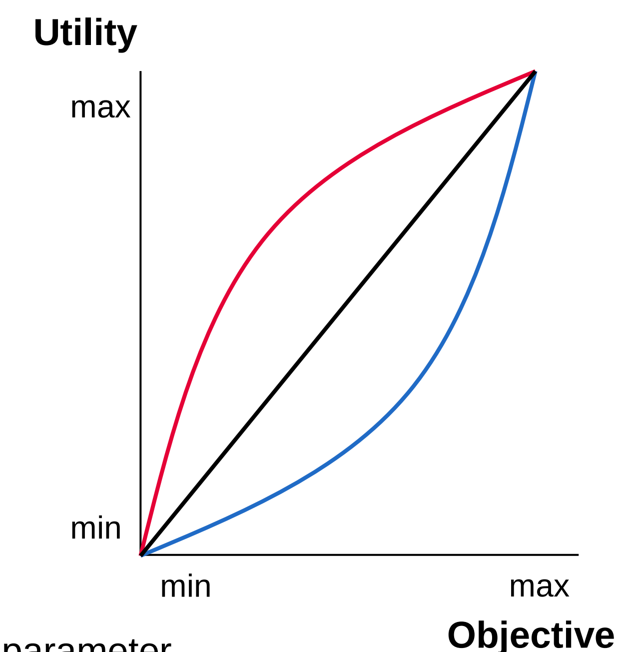
Introduction

- Managing forest resources according to a DM's attitudes towards risk
- Forest planning is a multiple criteria decision problem, taken under uncertainty.
- Risk attitudes are an additional set of criteria which should be included in the planning process.
- We propose a stochastic programming model to incorporate the DM's risk preferences.

Review on risk preferences

- Risk averse (red, $\lambda > 1$):
  - Willing to incur a penalty in the primary objective to mitigate deviations
- Risk neutral (black, $\lambda = 1$):
  - unwilling to incur a penalty to primary objective, however does still wants to minimize deviations
- Risk seeking (blue, $0 < \lambda < 1$):
  - Willing to incur substantial deviations to maximize primary objective



primary objective

risk parameter

soft constraints

$$\max z = \sum_{j=1}^{J} \sum_{k=1}^{K_j} E(c_{jki1}) x_{jk} - \frac{\lambda}{I} \sum_{i=1}^{I} \left(\sum_{t=1}^{T} (w_t^- d_{it}^- + w_t^+ d_{it}^+) \right)$$

Incorporating uncertainty into the planning problem

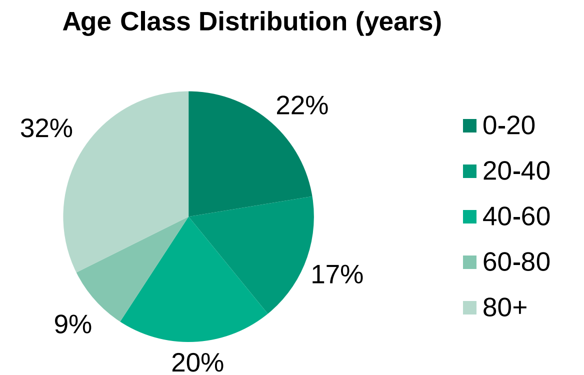
- Formulating a stochastic programming problem can be done through a deterministic approximation of the uncertainties. (Birge and Louveaux 2011)

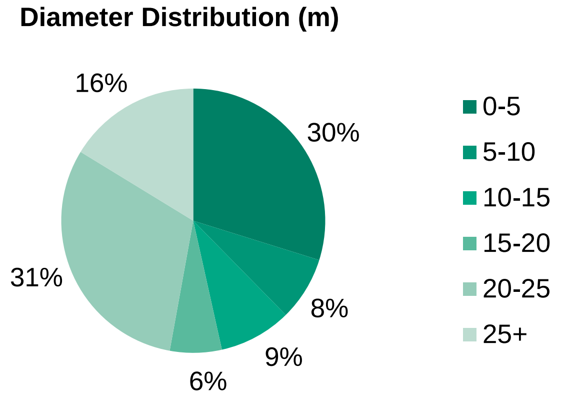
  - This requires the known (or estimated) distribution of the error.

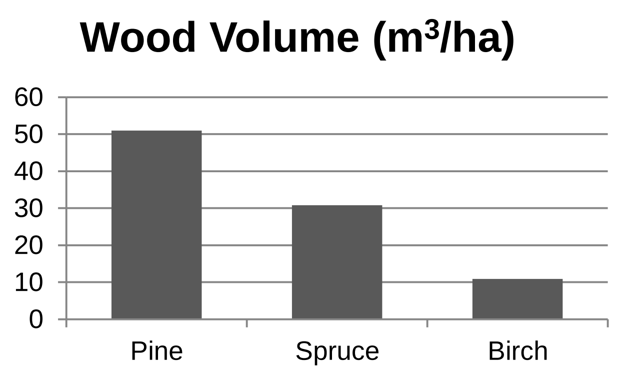
  - A number of scenarios are developed to approximate the distribution. (King and Wallace 2012)
    - A need for balance:
      - too many scenarios – tractability issues
      - too few scenarios – problem representation issues

Forest planning problem

- A forest where the DM wishes to
  - maximize first period income
    - subject to:
      - even flow constraints;
      - and an end inventory constraint.
  - Small forest holding
    - 47.3 hectares, 41 stands







Scenario generation approach:

- For this case, only the uncertainty from the inventory data was included.
  - A few assumptions were made:
    1. A recent inventory was conducted
    2. The inventory method was assumed to have an error which was normally distributed, mean zero and a standard deviation of 20% of the mean height and basal area.
- Using R, error was introduced into the inventory data, the process was repeated to generate 100 scenarios.
- Forest simulation was done using SIMO (Rasinmäki et al. 2009)
  - Created a set of 528 schedules for the 41 stands (~13 schedules per stand) for each scenario.

Formulations:

- Three formulations are developed:
  - The Expected Value problem (EV)
  - The Simple Recourse problem (RP)
  - The Wait-and-See problem (WS)

- The EV problem ignores uncertainty, and to make it comparable, the Expected result of the EV problem (EEV) needs to be calculated.

- With these results, Value of Information calculations can be done.
  - Expected Value of Perfect Information (EVPI) = RP-WS
  - Value of the Stochastic Solution (VSS) = EEV-RP

EV problem:
(case where original inventory data assumed correct)

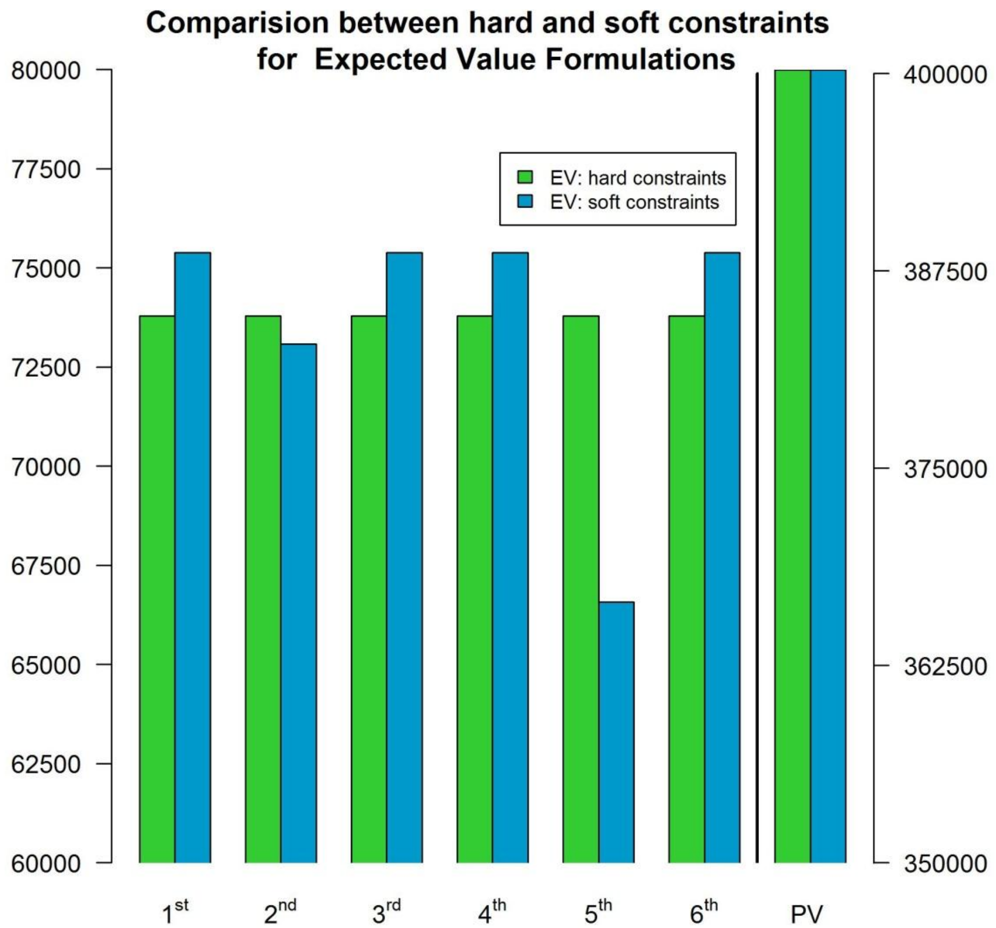
Objective Function :	
[1]	$\max EV = \sum_{j=1}^{J}\sum_{k=1}^{K_j} E(c_{jk1})x_{jk} - \lambda\left(\sum_{t=1}^{T}(p_t d_t^-) + p_E d_E^-\right)$
Subject to:	
[2a]	$\sum_{j=1}^{J}\sum_{k=1}^{K_j} E(c_{jkt})x_{jk} - \sum_{j=1}^{J}\sum_{k=1}^{K_j} E(c_{jk1})x_{jk} - d_t^+ + d_t^- = 0, \forall\, t = 2, \dots, T$ **Even-flow constraint**
[3]	$\sum_{j=1}^{J}\sum_{k=1}^{K_j} E(PV_{jkT})x_{jk} - d_E^+ + d_E^- = \sum_{j=1}^{J} E(PV_{j0})$ **End volume constraint**
[4]	$\sum_{k=1}^{K_j} x_{jk} = 1, \forall\, j = 1, \dots, J$ **area constraint**
[5]	$d_t^+, d_t^- \geq 0, \forall\, t = 2, \dots, T$
[6]	$d_E^+, d_E^- \geq 0$

non-negativity constraints

- J – number of stands
- K_j – number of schedules for stand j
- T - period
- c_{jkt} – income
- PV – Present Value
- x_{jk} – treatment decision
- p_t – penalty parameter
- λ - risk parameter (included to the EV problem for comparative purposes)
- d_t, d_E – deviations from target

Selection of Penalties:

- One option is the use of EV shadow prices:
  - A representation of the price the DM is willing to pay for even flow
  - The EV shadow prices could be viewed as an approximation of the penalties set by a risk neutral individual.



RP problem:

Minimization of downside risk

Objective Function :	
[7]	$\max RP = \sum_{j=1}^{J}\sum_{k=1}^{K_j} E(c_{jki1})x_{jk} - \frac{\lambda}{I}\sum_{i=1}^{I}\left(\sum_{t=1}^{T}(p_t d_{it}^{-}) + p_E d_{iE}^{-}\right)$
Subject to:	
[8]	$\sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jkit}x_{jk} - \sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jki1}x_{jk} - d_{it}^{+} + d_{it}^{-} = 0, \forall\ i = 1, \dots, I, t = 2, \dots, T$
[9]	$\sum_{j=1}^{J}\sum_{k=1}^{K_j} PV_{jkiT}x_{jk} - d_{iE}^{+} + d_{iE}^{-} = \sum_{j=1}^{J} PV_{j0i}, \forall\ i = 1, \dots, I$
[10]	$d_{it}^{+}, d_{it}^{-} \geq 0, \forall\ i = 1, \dots, I, t = 2, \dots, T$
[11]	$d_{iE}^{+}, d_{iE}^{-} \geq 0, \forall\ i = 1, \dots, I$

- *I* – is the number of scenarios
- Rather than minimizing the expected weighted deviations, this formulation minimizes the sum of weighted negative deviations over all scenarios.
  - (i.e. Krzemienowski and Ogryczak 2005)

WS problem:

[12]	$\mathrm{WS} = \sum_{i=1}^{I} \frac{Z_i}{I}, \forall\, i = 1, \dots, I$
Objective Function (For all *i*=1,...,I , equations [13], [14], [15], and [16]):	
[13]	$\max z_i = \sum_{j=1}^{J} \sum_{k=1}^{K_j} c_{jki1} x_{jki} - \lambda \sum_{t=1}^{T} (p_t d_{it}^-) + p_E d_{iE}^-$
Subject to:	
[14]	$\sum_{j=1}^{J} \sum_{k=1}^{K_j} c_{jkit} x_{jki} - \sum_{j=1}^{J} \sum_{k=1}^{K_j} c_{jki1} x_{jki} - d_{it}^+ + d_{it}^- = 0, \forall\ t = 2, \dots, T$
[15]	$\sum_{j=1}^{J} \sum_{k=1}^{K_j} PV_{jkiT} x_{jki} - d_{iE}^+ + d_{iE}^- = \sum_{j=1}^{J} PV_{j0i}$
[16]	$\sum_{k=1}^{K_j} x_{jki} = 1, \forall\, j = 1, \dots, J$
	and subject to eqs [10] and [11].

- For each scenario an optimal set of schedules are selected.

Expected result of the EV problem:
(case where original inventory data assumed correct)

To calculate the EEV solution, let us denote $\bar{x}_{jk}$ as the optimal solution to [1]:

[17] $$EEV = \sum_{j=1}^{J}\sum_{k=1}^{K_j} E(c_{jki1})\bar{x}_{jk} - \frac{\lambda}{I}\sum_{i=1}^{I}\left(\sum_{t=1}^{T}(p_t \bar{d}_{it}^{-}) + p_E \bar{d}_{iE}^{-}\right)$$

Subject to:

[18] $$\bar{d}_{it}^{-}\begin{cases} 0\ if \sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jki1}\bar{x}_{jk} < \sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jkit}\bar{x}_{jk},\ i = 1, \dots, I, t = 2, \dots, T, \\ \sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jki1}\bar{x}_{jk} - \sum_{j=1}^{J}\sum_{k=1}^{K_j} c_{jkit}\bar{x}_{jk},\ i = 1, \dots, I, t = 2, \dots, T, otherwise \end{cases}$$

A vector of negative deviations

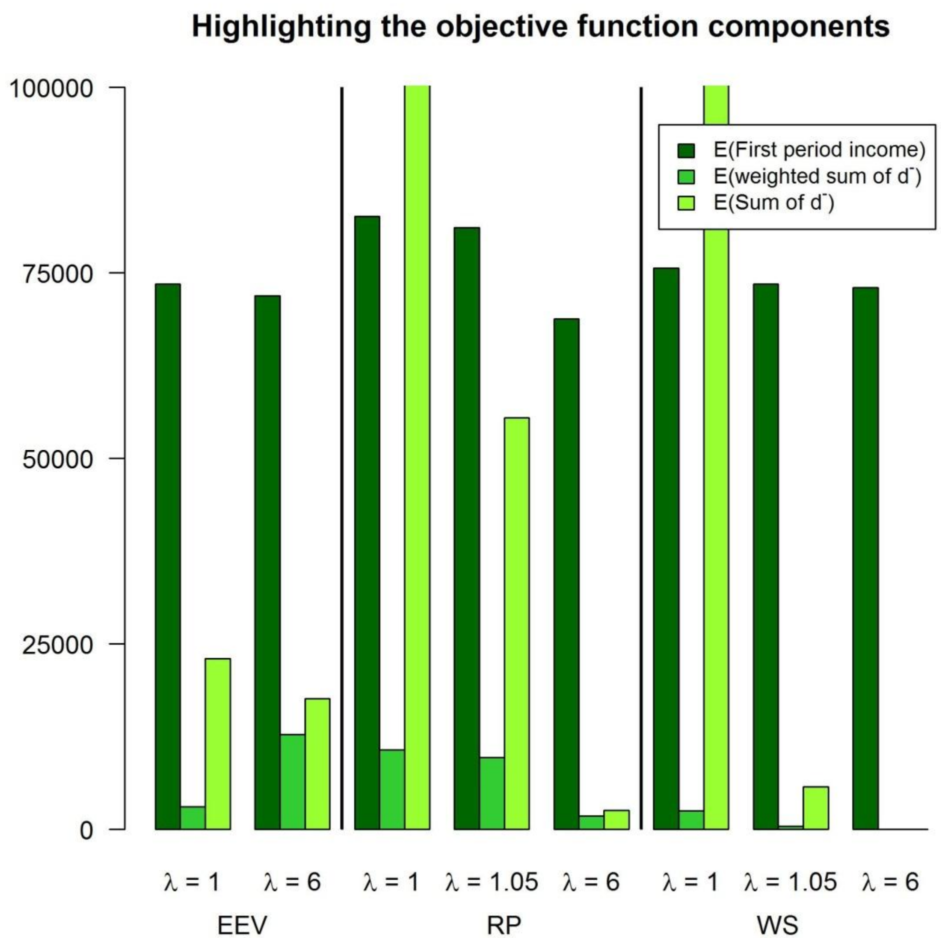
[19] $$\bar{d}_{iE}^{-}\begin{cases} 0\ if \sum_{j=1}^{J} PV_{j0i} < \sum_{j=1}^{J}\sum_{k=1}^{K_j} PV_{jkiT}\bar{x}_{jk},\ i = 1, \dots, I \\ \sum_{j=1}^{J} PV_{j0i} - \sum_{j=1}^{J}\sum_{k=1}^{K_j} PV_{jkiT}\bar{x}_{jk},\ i = 1, \dots, I, otherwise \end{cases}$$

End volume constraint

and subject to eqs [10] and [11].

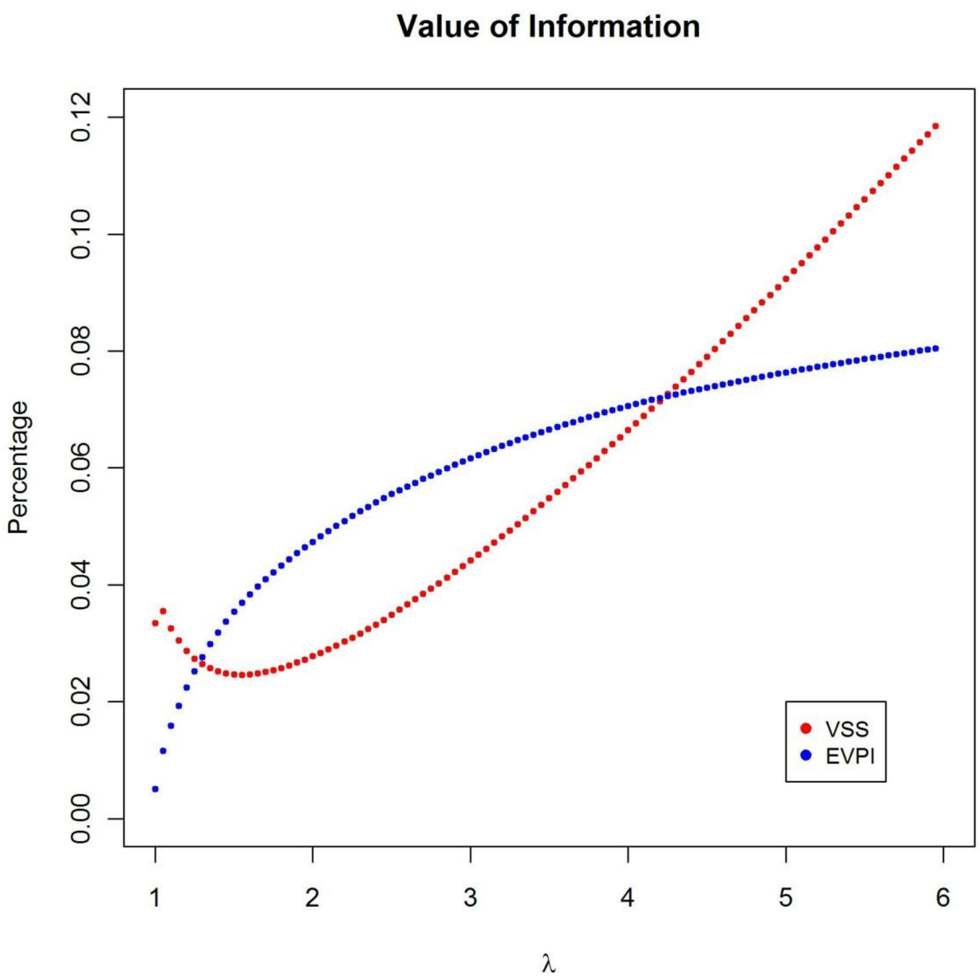
Results:

Separating primary objective and soft constraints



- Table separates the formulation into the components.
- For RP and WS problems, as λ increases,
  - Sum of negative deviations decrease,
  - 1st period expected income also decreases.
- Little change in the negative deviations for the EEV problem.
- Difference between $\lambda = 1$ and $\lambda = 1.05$
  - Penalty is only an approximation

Value of Information:



- Value of the Stochastic Solution
  - between 2.5 and 11.8%
  - $\lambda = 1$, slightly lower than $\lambda = 1.05$, due to approximation of penalties
  - The initial decrease is due 1st period income decreasing more rapidly than the weighted improvement of negative deviations
- Expected Value of Perfect Information
  - between 0.5 and 8%

Use of Shadow prices as penalties:

- Why the large spike in deviations?
  - The shadow prices for the RP problem do not precisely represent the 'risk neutral' weighting scheme
    - Same issue with the WS problem
      - If the shadow prices for each scenario were used, then for this example there would only be 2 solutions, when $\lambda = 1$, and when $\lambda > 1$

- Determining appropriate penalties is DM specific...

Conclusions:
(Methodological)

- VOI highlights the importance of including uncertainty into the problem formulation.

- Attitudes towards risk is preferential information, and is dependent on the individual.

  - This research focused on risk neutral and risk averse, risk seeking may be better served by more appropriate constraints...

Conclusions:
(Why is this important for the DM?)

- Using the RP formulation allows the DM a tool for managing risk
- In this case, the risk could be considered the regret of not achieving the same result as the first period.
  - The λ allows the DM to indicate how important this risk is.
- Allows the development of a plan which integrates more of the owners preferences.
- The RP solution is also simply more robust...

Future research

- Include further sources of error – growth models, climate change…
- Different criteria – this was a simple example, using economic goals, however it may be more useful in a more diverse multi-criteria setting.
  - This may make it easier to develop realistic risk models – i.e. how to balance the maximization of profit, with the risks of not reaching specific ecosystem service targets.
- Larger holdings.
- Multi-stage problems, with recourse.
  - add inventory measurements (i.e. Kangas et al. 2013)

References:

- Birge, J.R., and Louveaux, F. 2011. Introduction to stochastic programming. Second edition. Springer, New York. 499 p.
- Kangas, A., Hartikainen, M., and Miettinen, K. 2013. Simultaneous optimization of harvest schedule and measurement strategy. Scand. J. Forest Res.(ahead-of-print), 1-10. doi: 10.1080/02827581.2013.823237.
- King, A.J., and Wallace, S.W. 2012 Modeling with Stochastic Programming, Springer, New York
- Krzemienowski, A. & Ogryczak W. 2005. On extending the LP computable risk measures to account downside risk. Computational Optimization and Applications 32:133-160.
- Rasinmäki, J., Mäkinen, A., and Kalliovirta, J. 2009. SIMO: an adaptable simulation framework for multiscale forest resource data. Comput. Electron. Agric. **66**(1): 76–84. doi: 10.1016/j.compag.2008.12.007.